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(1, 2) - Triple Domination in Graphs

P.Murugaiyan[1], Dr.Paul Dhayabaran[2] and P.Padma[3]

[1,2]*Department of Mathematics, Bishop Heber College, Trichy, Tamilnadu, India. Email: npmuru@gmail.com, dpdhaya@yahoo.com*
[3]*Department of Mathematics, PRIST University, Thanjavur, Tamilnadu, India. Email: padmaprithivirajan@gmail.com*

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ABSTRACT

Berge [1] and Ore [7] were the first to define dominating sets. A new type of dominating set, (1, 2) – dominating set is introduced by Steve Hedetniemi and Sandee Hedetniemi [8]. In this paper, we introduced the (1, 2) – triple domination number and also we discussed about its properties.

Keywords: (1, 2) - triple domination number.

1. INTRODUCTION

Dominating queens is the origin of the study of dominating set in graphs. Berge [1] and Ore [7] were the first to define dominating sets. A new type of dominating set, (1, 2) - dominating set is introduced by Steve Hedetniemi and Sandee Hedetniemi [8].

In this paper, we have introduced the (1, 2) - triple domination number and also we discussed about its properties.

2. PRELIMINARIES

Definition 2.1: A graph is said to be complete if each of its vertices is adjacent to every other vertex.

Definition 2.2: A graph is said to be regular if each of its vertices has the same degree.

Definition 2.3: A graph is said to be cubic graph if each of its vertices is of degree three.

Definition 2.4: A bipartite graph is a graph in which vertices can be divided into two disjoint sets A and B such that every edge connects a vertex in A to one in B.

Definition 2.5: A (1, 2) – dominating set in a graph G = (V,E) is a set S having the property that for every vertex v in V – S there is atleast one vertex in S at distance 1 from v and a second vertex in S at distance atmost 2 from v.

Definition 2.6: The order of the smallest (1,2)- dominating set of G is called the (1,2) – domination number of G and we denote it by γ (1,2) .

Remark 2.1: From the definition of 2.1, we see that a (1,2) – dominating set contains atleast 2 vertices, (1,2) – domination number of a graph will be always ≥ 2 and (1,2) – dominating sets occur in graphs of order atleast 3.

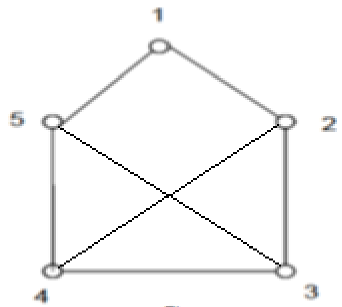
Definition 2.7: For each vertex x in a graph G, we introduce a new vertex x' and join x and x' by an edge. The resulting graph is called the ***corona*** of G.

3. (1, 2) – TRIPLE DOMINATING SET

Definition 3.1 : A (1, 2) – triple dominating set in a graph G = (V, E) is a set S having the property that for every vertex v in V – S there is atleast three vertex in S at distance 1 from v and a second vertex in S at distance atmost 2 from v.

Definition 3.2: The order of the smallest (1, 2) - triple dominating set of G is called the (1, 2) – triple domination number of G and we denote it by $\gamma_{3d(1,2)}$. From the definition of (1,2) – triple dominating sets, we see that a (1, 2) – triple dominating set contains atleast 2 vertices, (1,2) – triple domination number of a graph will be always ≥ 3 and (1, 2) – triple dominating sets occur in graphs of order atleast 3.

Example 3.1: Consider the graph



G

In G, {1, 4, 3, 2} is a (1, 2) – triple dominating set.

Definition 3.3: A dominating set S is an independent triple dominating set if no two vertices in S are adjacent, that is, S is an independent set. The independent triple domination number $i_3(G)$ of a graph G is the minimum cardinality of an independent triple dominating set. Thus $i_3(G)$ = min{ | S | dominates and $\Delta(\langle S \rangle)$} .

Definition 3.4 : A triple dominating set S is called a perfect triple dominating set if for every vertex , The perfect triple domination number is denoted as $u \in V$, $| N [u] \cap S | = 1$. The perfect triple domination number is denoted as $\gamma_{pt}(G)$.

Definition 3.5 : A triple dominating set S is called an efficient triple dominating set if for every vertex, $u \in V - S$,

|N(u) ∩ S|= 1. Equivalently, a triple dominating set is efficient if the distance between any two vertices in S is at least three, that is, S is a packing. We note that, if a graph has an efficient triple dominating set, then all efficient triple dominating sets in G have the same cardinality namely γ(G).

Theorem 3.1: All (1, 2) – triple dominating sets are dominating sets.

Proof: The result is trivial from the definition of (1, 2) – triple dominating sets.
But the converse need not be true.

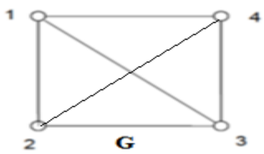
Example 3.2: In example 3.1, { 1,4} is a dominating set . But it is not a (1, 2) – dominating set. { 2, 3, 4 } is a (1, 2) – dominating set . { 1 , 4, 3 } is a (1, 2) – triple dominating set and it is a dominating set also.

3.2. (1, 2) – Triple Domination In Complete Graphs
Theorem 3.2.1 : (1, 2) – triple domination is not possible in complete graphs.

Proof: In a complete graph, each vertex is adjacent to every other vertices. So we cannot find a (1, 2) – triple dominating set. No vertex can be found at a distance atmost 2 from any other vertex. Let G be a complete graph with n vertices. Then it will have nC2 edges and each vertex is of degree n – 1.The minimum number of edges to be deleted so as to become the resulting graph (1, 2) – triple dominating is n - 2 . If we delete n – 2 edges from a complete graph, then in the resulting graph, we can find a (1, 2) – triple dominating set.

Lemma 3.2.1 : If a graph G with n vertices, has a vertex of degree n – 1 ,we cannot find a (1, 2) – dominating set.

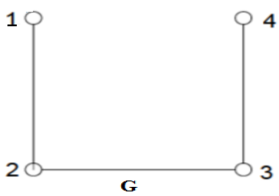
Example 3.2.1 : In this graph, we cannot find a (1, 2) – triple dominating set since each vertex is adjacent to all other vertices.

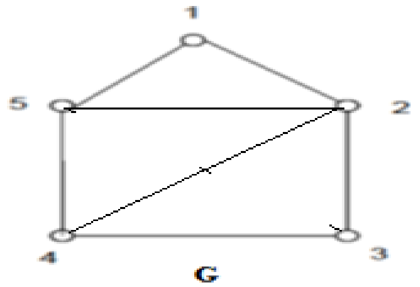


In graph G , we cannot find a (1, 2) – triple dominating set since each vertex is adjacent to all other vertices.

3.3. Relation Between Domination Number And (1, 2) – Triple Domination Number
In this section we consider different types of graphs and find out their domination number , (1, 2) - domination number and (1, 2) – triple domination number and check the relation between them.

Example 3.3:

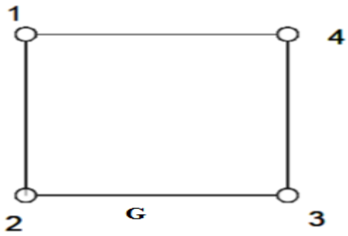




In G, {1,3}, {1,4}, {2,4}, {2,3} are all dominating sets. γ (G) = 2.
{1, 4} is a (1, 2) – dominating set.
γ (1,2) = 2.
{1, 3, 4} is a (1, 2) – triple dominating set.

$\gamma_{d(1,2)} = 3$.
$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.
$\gamma < \gamma_{d(1,2)}$.

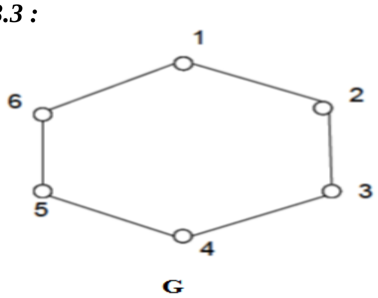
Example 3.3.2:



In G,
{1, 3}, {1, 4}, {2, 4}, {2, 3} are all dominating sets.
γ (G) = 2.
{2, 3} is a (1, 2) – dominating set.
γ (1,2) = 2.
{2, 3, 4} is a (1, 2) – triple dominating set.

$\gamma_{d(1,2)} = 3$.
$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.
$\gamma < \gamma_{d(1,2)}$.

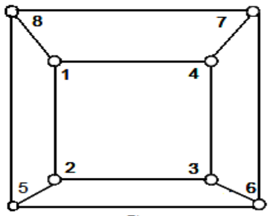
Example 3.3.3 :



In G ,
{1, 3, 5}, {2, 4, 6} are dominating sets.
γ (G) = 3.
{1, 4, 6} is a (1, 2) – dominating set.
γ (1,2) = 3.
{1, 3, 4, 6} is a (1, 2) – triple dominating set.

$\gamma_{d(1,2)} = 4$
$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.
$\gamma < \gamma_{d(1,2)}$.

Example 3.3.4:



G

In G,

{1,2,3,4} , {5,6,7,8} are dominating.

γ (G) = 4 .

{1,2,3,4} is a (1,2) – dominating set.

γ (1,2) = 3.

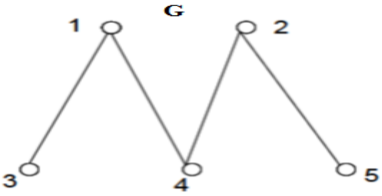
{ 1, 3, 5, 6, 7, 8 } is a (1, 2) – triple dominating set.

$\gamma_{d(1,2)} = 6$

$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.

$\gamma < \gamma_{d(1,2)}$.

Example 3.3.5: Consider the bipartite graph G



In G,

{1,2} is a dominating set.

γ (G) = 2.

{1,4,5} is a (1,2) – dominating set.

γ (1,2) = 3.

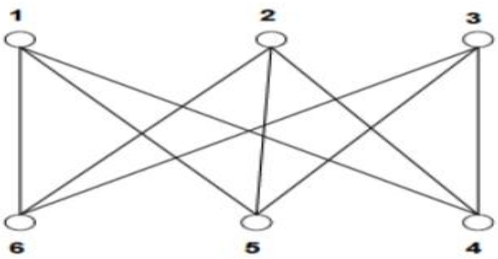
{ 2, 3, 4, 5 } is a (1, 2) – triple dominating set.

$\gamma_{d(1,2)} = 4$

$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.

$\gamma < \gamma_{d(1,2)}$.

Example 3.3.6: Consider the cubic bipartite graphs G,



G

In G,

{ 1, 5 } , { 2, 6 } is a dominating set.

γ (G) = 2.

{ 1, 5 } is a (1,2) – dominating set.

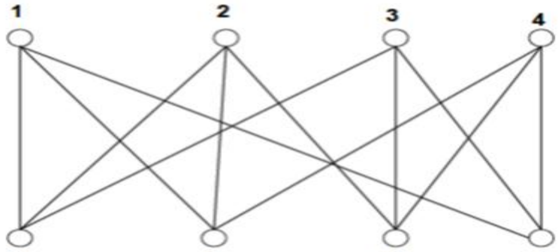
γ (1,2) = 2.

{ 2, 4, 6, 5 } is a (1, 2) – triple dominating set.

$\gamma_{d(1,2)} = 4$

$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.

$\gamma < \gamma_{d(1,2)}$.



G

In G,

{1, 6} is a dominating set.

γ (G) = 2.

{1, 6} is a (1,2) - dominating set.

γ (1,2) = 2.

{1, 3, 6, 7, 8} is a (1, 2) - triple dominating set.

$\gamma_{d(1,2)} = 5$.

$\therefore \gamma(1,2) < \gamma_{d(1,2)}$.

$\gamma < \gamma_{d(1,2)}$.

Remark 3.3.1: In all the above examples, we conclude the following,

- domination number is less than (1, 2) – triple domination number.
- (1, 2) – domination number is less than (1, 2) – triple domination number.

From the above examples we have the following theorem.

Theorem 3.3.1: In a graph G, domination number is less than (1, 2) – triple domination number.

Proof: Let G be a graph and D be its triple dominating set. Then every vertex in V – D is adjacent to a vertex in D. That is, in D, for every vertex u, there is a 2 vertex which is at distance 1 from u. But it is not necessary that there is a second vertex at distance atmost 2 from u. So if we find a (1, 2) – dominating set, it will contain more vertices or atleast equal number of vertices than the dominating set. So the domination number is less than (1, 2) – domination number.

Theorem 3.3.2: In a graph G, (1, 2) – domination number is less than or equal to (1, 2) – triple domination number.

Proof: Similar to theorem 3.3.1.

Theorem 3.3.3: If G is a 2-regular graph, then the (1, 2) – triple domination number of the corona of G is equal to the number of vertices of G.

Proof: Let G be a 2- regular graph. Then each of its vertices will be of degree 2. In the corona of G, for each vertex x, we introduce a new vertex and join them. Consequently, an edge is added to each of its vertices. By the definition of (1,2) – triple dominating set each vertex v in V – S has atleast two vertex in S at distance 1 from v and a second vertex in S at distance atmost 2 from v. Hence (1,2) – triple dominating set of the corona of G will consist of all the vertices of G.

Theorem 3.3.4: If in a graph G, an edge e is added, $\gamma_{d(1,2)}(G+e) \geq \gamma_{d(1,2)}(G)$.

Proof: Let G be a graph. Let S be the (1, 2) – triple dominating set of G. If we add an edge to a vertex in S, that will not affect the cardinality of S. If we add an edge to a vertex in V- S , the cardinality of (1, 2) – triple dominating set will increase. Therefore, $\gamma_{d(1,2)}(G+e) \geq \gamma_{d(1,2)}(G)$.

Theorem 3.3.5 : If G is a complete bipartite graph, then the (1, 2) – triple domination number $\gamma_{d(1,2)}$ is 3.

Proof: Let G be a complete bipartite graph. Then V(G) can be partitioned in to 2 disjoint sets X and Y and each edge has one end in X and other end in Y. Since G is complete bipartite, each vertex of X is joined to every vertex in Y. A set of 2 vertices, one from X and another from Y will constitute a (1, 2) – triple dominating set. Therefore, $\gamma_{d(1,2)}$ = 3.

4. CONCLUSION

We considered the problem of finding a (1, 2) - triple dominating set in graphs and compared them with the domination number. Also some preliminary theorems on (1, 2) - dominating sets are proved.

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