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Determining the Indeterminate Edges by α-cut Coloring Method

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ABSTRACT

An uncertain graph is a graph in which the edges are indeterminate and the existence of edges is characterized by belief degrees which are uncertain measures. The coloring of regular fuzzy graph G=(V,B) is introduced in previous works. The two different approaches are applied to coloring of Regular fuzzy graph. The first approach is based on the odd regular coloring fuzzy graphs and the second approach is based on the even regular coloring fuzzy graph. We establish strong coloring of a fuzzy graph and we change $\delta -$ arc into $\alpha -$ arc or $\beta -$ arc. This work aims to bring graph coloring and uncertainty theory together. A new approach for uncertain graph coloring based on a α-cut of an uncertain graph is introduced in this work. Firstly, the concept of α-cut of uncertain graph is given an some of its properties are explored. By means of α-cut coloring, we get a α-cut chromatic number and examine some of its properties as well. Then, a fact that every α-cut chromatic number may be a chromatic number of an uncertain graph is obtained, and the concept of uncertain chromatic set is introduced. In addition, an uncertain chromatic algorithm is constructed. Finally, a real-life decision making problem is given to illustrate the application of the uncertain chromatic set and the effectiveness of the uncertain chromatic algorithm.

Keywords: Regular fuzzy graphs, graph coloring, strong coloring, chromatic number, uncertainty.

1. INTRODUCTION

In mathematics, graph theory is the study of graphs, which are mathematical structures used to model pair wise relations between objects. A graph in this context is made up of vertices, nodes, or points which are connected by edges, arcs, or lines. A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be directed from one vertex to another [2].

Graphs can be used to model many types of relations and processes in physical, biological, social and information systems. Many practical problems can be represented by graphs. Emphasizing their application to real-world systems, the term network is sometimes defined to mean a graph in which attributes (e.g. names) are associated with the nodes and/or edges.

In computer science, graphs are used to represent networks of communication, data organization, computational devices, the flow of computation, etc. For instance, the link structure of a website can be represented by a directed graph, in which the vertices represent web pages and directed edges represent links from one page to another. A similar approach can be taken to problems in social media, travel, biology, computer chip design, mapping the progression of neuro-degenerative diseases, and many other fields. The development of algorithms to handle graphs is therefore of major interest in computer science. The transformation of graphs is often formalized and represented by graph rewrite systems. Complementary to graph transformation systems focusing on rule-based in-memory manipulation of graphs are graph databases geared towards transaction-safe, persistent storing and querying of graph-structured data [4].

Graph-theoretic methods, in various forms, have proven particularly useful in linguistics, since natural language often lends itself well to discrete structure. Traditionally, syntax and compositional semantics follow tree-based structures, whose expressive power lies in the principle of compositionality, modeled in a hierarchical graph. More contemporary approaches such as head-driven phrase structure grammar model the syntax of natural language using typed feature structures, which are directed acyclic graphs.

The vertex coloring problem is the problem of selecting a color to each vertex of a graph such that the colors of adjacent vertices are dissimilar and the number of colors needed is minimized. A lot of researchers have investigated methods on how to color a graph with the minimum number of colors. The vertex coloring problem can be used to model many realistic application problems. In such applications, the vertices of a graph represent items of interest and the vertex coloring problem presents the challenge of grouping the items in as few groups as possible subject to the constraint that incompatible items end up in the different groups.

2. RELATED WORK

The reader may refer to Brown (1972) for more detail applications of graph coloring. We refer to the classical graph as a crisp graph. The edges and vertices of the crisp graph are all stated definitely [1]. However, the real-life situations which are represented by a graph may be so complex such that adjacency between two vertices cannot be completely determined. Such a system cannot be modeled by a crisp graph. Therefore, we need a tool to represent these indeterminate phenomena in the field of graph theory.

Zadeh (1965) dealt with the indeterminate phenomena within the framework of fuzzy set theory. Since then, the concept of fuzzy graph was proposed by several authors to handle the fuzzy phenomena in graphs [2]. Muñoz et al. (2005) put forth the method for fuzzy graph coloring in two different ways [3]. In the first approach, the coloring of a fuzzy graph was investigated by using classical coloring on an α-cut of a fuzzy graph. In the second approach, the coloring of a fuzzy graph was done by using a coloring function, which depends upon the distance defined between two colors. The main goal of this paper is to introduce a new approach for coloring an uncertain graph based on an α-cut of the uncertain graph. We introduce a concept of an α-cut of an uncertain graph, and investigate some of its properties. By means of α-cut coloring, we obtain an α-cut chromatic number and investigate some properties of the α-cut chromatic number. We find that every α-cut chromatic number may be a chromatic number of an uncertain graph. Hence, we have an uncertain chromatic set of an uncertain graph. Finally, an uncertain chromatic algorithm is constructed and an illustration for the application of uncertain chromatic set is given. The contributions of this paper are to establish mathematical theory on uncertain graph coloring, to give a new method for uncertain graph coloring which is a useful tool for decision making on uncertain environment, and to develop an algorithm for uncertain graph coloring.

3. UNCERTAIN THEORY

Uncertain Graph is a generative model for deterministic graphs. A deterministic graph G is generated by Ĝ by connecting two nodes u, v via an edge with probability P_uv. The probability that G=(V, E) sampled from Ĝ = (V, P)) is:

$$\Pr[G] = \prod_{\{u,v\}\in E_G} P_{uv} \prod_{\{u,v\}\in (V\times V)\setminus E_G} (1 - P_{uv}).$$

Uncertain graphs are employed to describe graph models with in deterministic information that produced by human beings. This work aims to study the maximum matching problem in uncertain graphs [6]. The number of edges of a maximum matching in a graph is called matching number of the graph. Due to the existence of uncertain edges, the matching number of an uncertain graph is essentially an uncertain variable. Different from that in a deterministic graph, it is more meaningful to investigate the uncertain measure that an uncertain graph is k-edge matching (i.e., the matching number is greater than or equal to k). We first study the properties of the matching number of an uncertain graph, and then give a fundamental formula for calculating the uncertain measure Uncertainty theory is another mathematical system to model subjective uncertainty. For the scientists of uncertainty theory, uncertainty theory offers a powerful tool to deal with the human's belief degree. The indeterminate quantity "potential friends" in a relationship network can be assumed to be an uncertain variable. Uncertainty theory was founded by Liu in 2007 and refined by Liu in 2010.

Definition: Let Γ be a nonempty set. A collection L of a subset of Γ is called an algebra over Γ if it satisfies: (a) Γ ∈ L; (b) if Λ ∈ L, then Λc ∈ L;

Uncertain Graph: In this subsection, some basic terminology of graph theory refers to Bondy and Murty and Gibbons. Assume V is a set of vertices, and E is a set of edges. Then a graph can be denoted by G = (V, E). A matching H in a graph G is essentially a subset of E such that every vertex is incident to at most one edge in the set. If a matching H satisfies |H| ≥ |H0 | for any matching H0 , then H is said to be maximum, where |H| denotes the number of edges in H. Figure 1 illustrates two classic graphs. Graph G1 contains a matching {e1}, which is also the maximum matching. Graph G2 has an edge set {e1, e2, e3}. We can obtain a maximum matching {e1, e3} by removing e2. The number of edges of a maximum matching in a graph is said to be matching number of the graph. In this paper, we denote by α(G) the matching number of a graph G. The matching number in a classic graph is deterministic. However, it is not the case in a graph in the state of indeterminacy. Based on fuzzy set theory, the concept of fuzzy graph was proposed [8]. A standardized definition of fuzzy graph can be referred as follows. Definition: (Mordeson and Nair)A fuzzy graph G = (V, μ, ρ) is a nonempty set V together with a pair of functions μ : V → [0, 1] and ρ : V × V → [0, 1] such

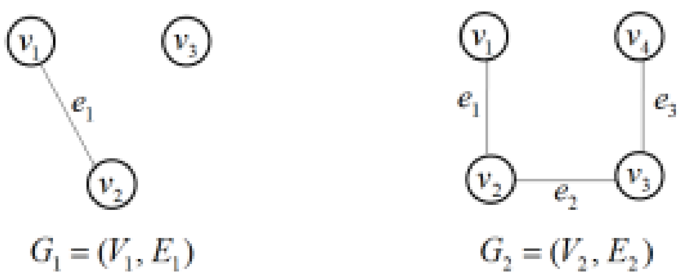

Figure No: 1 Classic Graphs

that ρ(x, y) ≤ μ(x) ∧ μ(y) for all x, y in V , where μ is a fuzzy subset of V and ρ is a symmetric fuzzy relation on μ. Based on uncertainty theory, the concept of uncertain graph was defined by Gao and Gao in 2013.

4. CLASSICAL GRAPH THEORY

In this subsection, the definition of graph and basic terminologies of graph theory are reviewed in a more general setting. We assume that every graph is an undirected, simple, and finite graph. Definition: (Bondy and Murty 1976) A graph G is an ordered triple (V, E, ψG) consisting of a nonempty vertex set V(G), an edge set E(G) and an incidence function ψG that associates with each edge of G an unordered pair of vertices of G. The number of vertices in the graph G = (V, E, ψG) is often called the order of G [5]. If e is an edge, and u and v are vertices such that ψG(e) = (u, v), then the vertices and v are called adjacent. A set I ⊆ V is said to be an independent vertex set if (u, v) /∈ E(G) for all u, v ∈ I. Given a graph G = (V, E, ψG). A k-coloring of the graph G can be defined through two approaches. In the first approach, the k-coloring of the graph G is defined as a mapping from the vertex set V into the set of colors{1, 2,..., k}such that two adjacent vertices are assigned with different colors. In the second approach, the k-coloring of the graph G is defined as a partition of V into k-independent vertex sets V1, V2,..., Vk such that k i=1 Vi = V where the subsets Vi are nonempty, and Vi ∩ Vj = ∅ for i = j. The minimum number k needed in the k-coloring of G, is called the chromatic number of G, denoted by χ (G).

5. A REGULAR FUZZY GRAPH COLORING AND UNCERTAINTY THEORY

An arc (u,v) is said to be strong if μ(u, v) ≥ Conn(u,v). The Strong arcs are classified as (i)α − strong arc (ii) β − strong arc (iii)δ − strong arc.

Definition: α − strong arc An arc is said to be α − strong if μ(u,v) > Conn(u,v)

Definition: β − strong arc An arc is said to be β − strong if μ(u,v) = Conn(u,v)

Definition: δ − strong arc An arc is said to be δ − strong if μ(u,v) < Conn(u,v)

Definition: The strength of connectedness between two nodes x and y,is defined as the maximum of strength of all paths between x and y, and it is denoted by conn (x, y) G .

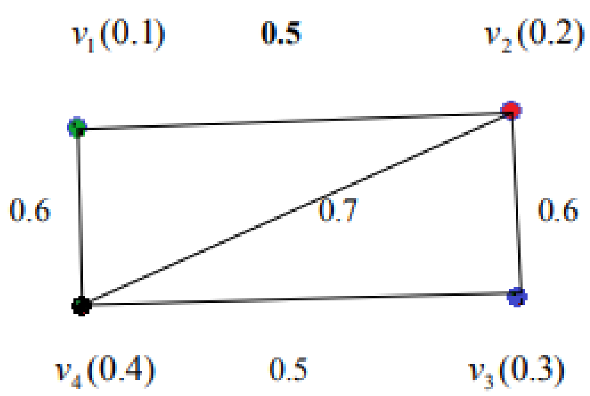

Figure No: 2 A Regular Fuzzy Graphs

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Theorem: If the graph G = (A,B) is odd regular strong arc fuzzy graph if each vertex has distinct color. Proof: Let G = (A, B) be a fuzzy graph on G (: A,B) ∗ .If d(v) = k∀v ∈V .That is if each vertex has same degree k ,then G is regular fuzzy graph. If the number of vertex is odd number then the graph is said to be odd regular graphs otherwise is said to be even regular graphs. An arc is said to be strong if μ(u, v) ≥ Conn(u,v).Let { , ,....., } V = V1 V2 Vn .We assume that G is odd (or) even regular strong arc fuzzy graph. To Prove: If all the vertex has distinct color. Every vertex is adjacent to other vertex. By our definition no vertex is adjacent to any vertex in that case the vertex has same color. Otherwise each vertex has distinct color. Conversely, we assume that the graph G has distinct color. To Prove: The graph is odd (or) even regular strong arc fuzzy graph [7]. By our definition of coloring { , ,....., } of fuzzy sets on a set V is called a k −fuzzy coloring of G = (V ,σ ,μ) if (i) ∨ Γ = σ (ii)γ i ∧ γ j = 0.

6. COLORING AN UNCERTAIN GRAPH VIA ITS A-CUT

This section will introduce the concept of an α-cut of an uncertain graph and investigate several properties of the new concept.

Definition: Let α ∈ [0, 1]. An α-cut of uncertain graph Gˇ = (V, E,ξ) is a crisp graph Gα = (V, Eα), where Eα = {(vi, v j)|vi, v j ∈ V, M{ξi j = 1} ≥ α}. An example that illustrates the concept of the α-cut of an uncertain graph is given as follows. Consider the uncertain graph Gˇ = (V, E,ξ). There are five vertices in the uncertain graph Gˇ and its edges exist with the corresponding belief degrees 0.1, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, and 1, respectively. Further, some properties of the α-cut of an uncertain graph are investigated as follows.

Theorem: If Gˇ = (V, E,ξ) be an uncertain graph with n vertices and G∗ be the underlying graph of G, then the ˇ α-cut Gα = G∗ for all 0 < α ≤ min{M{ξi j = 1}|i, j = 1, 2,..., n}.

Proof Let αr = min{M{ξi j = 1}|i, j = 1, 2,..., n}, and 0 < α ≤ αr. The edge set Eα = {(vi, v j)|vi, v j ∈ V, α ≤ M{ξi j = 1} < αr}∪{(vi, v j)|vi, v j ∈ V, M{ξi j = 1} ≥ αr}. Since αr = min{M{ξi j = 1}}, we get {(vi, v j)|vi, v j ∈ V, α ≤ M{ξi j = 1} < αr}=∅. Thus, Eα = {(vi, v j)|vi, v j ∈ V, M{ξi j = 1} ≥ αr}. It means that Eα contains all of pairs (vi, v j) in Gˇ where ξi j = 1, and M{ξi j = 1} > min{M{ξi j = 1}|i, j = 1, 2,..., n}. We consider that the underlying graph G∗ is obtained from the uncertain graph Gˇ by replacing the uncertain measure for each pair(vi, v j) where ξi j = 1, by M{ξi j = 1} = 1 for i, j = 1, 2,..., n. Therefore, we get each edge (vi, v j) ∈ Eα in Gα is contained in E∗ of G∗. Thus, Eα = E∗ and Gα = G∗ for all 0 < α ≤ min M{ξi j = 1} .

If Gˇ = (V, E,ξ) be an uncertain graph with n vertices and max{M{ξi j = 1}|i, j = 1, 2,..., n} < 1, then the α-cut Gα is an isolated graph for all max{M{ξi j = 1}|i, j = 1, 2,..., n} < α ≤ 1. Proof Since 1 ≥ α > max{M{ξi j = 1}|i, j = 1, 2,..., n}, Eα = ∅. In other words, the α-cut Gα = (V, Eα) is a graph without edges (isolated graph). Each α-cut of an uncertain graph is a crisp graph. We call the chromatic number of an α-cut of an uncertain graph as α-cut chromatic number, denoted by χα. Hence, coloring the uncertain graph Gˇ is transformed into classical coloring via the α-cut of Gˇ.

7. RESULTS AND DISCUSSION

In the crisp graph, the adjacency between two vertices is deterministic. However, in a real-life situation, some indeterminate factors will appear because of the lack of observation data, insufficient information and other reasons. In such cases, the adjacency between two vertices is not completely determined. Hence, we can- not handle such a situation by a crisp graph. However, by taking advantage of uncertainty theory, some indeterminate factors in the adjacency of vertices can be handled by uncertain variables.

Methods	Accuracy
Classical graph theory	87%
Uncertain and Chromatic theory	95%

Table No: 1 Accuracy Comparison

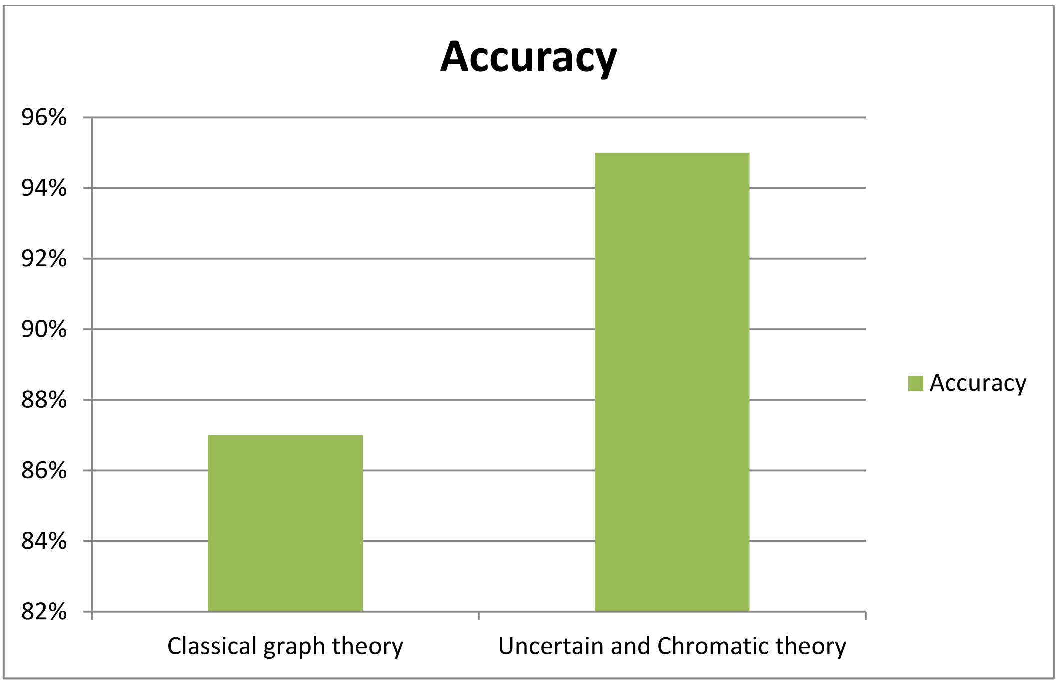

Figure No: 3 Accuracy Comparison

The accuracy comparison between regular fuzzy graphs and uncertain theory is given in table 1 and figure 3.

One of the coloring problems of a crisp graph G is to determine the chromatic number of G, which is a crisp number. In practical applications, as discussed in the previous sections, the crisp chromatic number cannot handle some of the indeterminate factors in a coloring problem. However, through uncertain graph coloring, we obtain an uncertain chromatic set which is a useful tool to handle uncertain factors in the coloring problem.

8. CONCLUSION

In this work, we proposed the concept of the α-cut of an uncertain graph in order to deal with uncertain graphs by means of crisp graphs. Next, some properties of the α- cut of an uncertain graph were investigated. One of the

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effective methods for solving the coloring problem in the nondeterministic graph is to transform it into classical coloring. Hence, we colored an uncertain graph via α-cut coloring and obtained α-cut chromatic number, we also discussed some properties of the α-cut chromatic number. Furthermore, we proposed an uncertain chromatic algorithm based on α-cut color- ing. The algorithm was decomposed into two steps. The first step was to find the α-cut chromatic number of an uncertain graph. The second step was to find the uncertain chromatic set of an uncertain graph. Any algorithm for computing the chromatic number of a crisp graph could be used to compute the α-cut chromatic number. Therefore, the uncertain chromatic algorithm proposed in this paper was simpler and easier than the algorithm based on a maximal uncertain independent vertex set.

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