

Calculating Degree Based Multiplicative Topological indices of Alcohol

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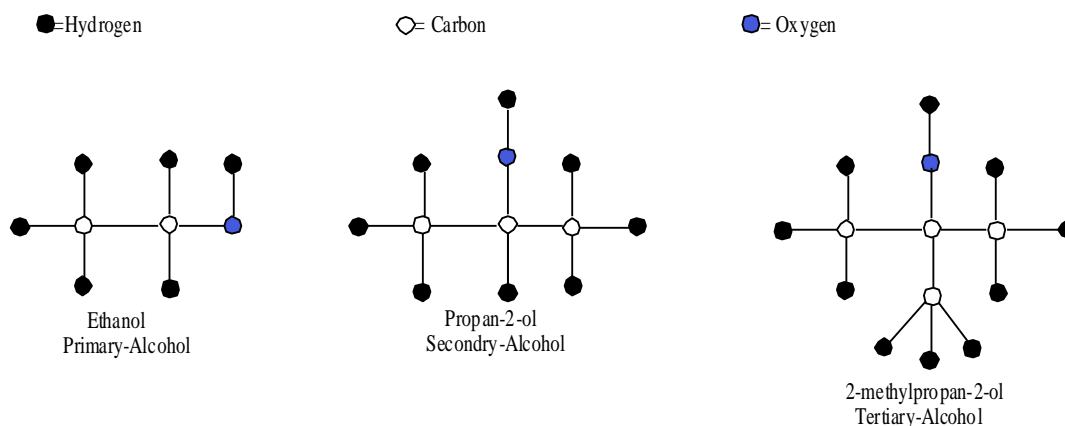
ABSTRACT

Description the entire structure of a graph by a mathematical quantity called topological index. These topological indices are categorized on the basis of their logical roots from topological invariants. Degree based topological indices depends upon the degree of the vertex. In papers [1, 2, 3] authors define some multiplicative topological indices on graphs. Throughout this article, we will work out on the various degree based topological indices of a functional group alcohol.

Keywords: Molecular graph theory, Molecular graph, Multiplicative Topological indices, Degree based topological indices, Degree (of a vertex), Structure of alcohol, Geodesic Metric Space.

1. INTRODUCTION

Alcohol particles are natural atoms that contain a $-OH$ group. This $-OH$ aggregate makes the atom interested, so it is known as a useful gathering. Alcohol practical gatherings are found in organic particles, for example sugars, amino acids and vitamins. Alcohol practical gatherings are additionally found in particles that are utilized each day. You can discover ethanol in drinks like wine, lager, vodka, bourbon, gin and rum, among numerous others. It is also use in the cars in the form of the antifreeze in the form of the ethylene glycol. Alcohol molecules can be classified based on the hydroxyl group $-OH$ attached. The locality of $-OH$ group can create difference among the different types of alcohols by that locality of $-OH$ group it may change the physical and chemical properties of an alcohol. There are mainly three types of alcohol. Alcohols are categorized as primary, secondary and tertiary due to different location of $-OH$ group in the structure of alcohol.



Alcohols are also exist in the form of cyclic alcohols. General formula for alcohols is $C_nH_{2n+1}OH$. For getting more information about alcohols, we strongly refer [4, 5, 6].

Now a days, the most cited and researching area of mathematics is graph theory especially chemical graph theory. In this paper we will find some multiplicative topological indices of alcohol. The main idea of this paper directly came from [1, 2, 3].

2. DEFINITIONS AND PRELIMINARIES

Let G be the molecular graph with $V(G)$ represents the set of vertices, $E(G)$ is the set of all edges between the vertices of the graph. Define a metric space [7] $d: V(G) \times V(G) \rightarrow R$, distance between $u, v \in V(G)$, $d_G(u, v)$ is define as the number of edges between u and v in shortest path, this metric space is called geodesic metric space. Let $j \in V(G)$ be a vertex then neighborhood [7] of j in graph G is define as $N_G(j) = \{x \in V(G) \mid d_G(j, x) = 1\}$ and cardinality of neighborhood set of j is called the degree [8] of j in graph G which represents as d_j .

Now we will define some degree based multiplicative topological indices of graphs. First and second Zagreb indices was define in the paper [9] and multiplicative Zagreb indices [10, 11] are define such as

$$II_1(G) = \prod_{u \in V(G)} (d_u)^2,$$

$$II_2(G) = \prod_{uv \in E(G)} d_u \cdot d_v.$$

In the paper [12] authors define a multiplicative version of first Zagreb index as

$$II_1^*(G) = \prod_{uv \in E(G)} (d_u + d_v).$$

In [13] G. H. Shirdel, H. Rezapour, and A. M. Sayadi define the hyper-Zagreb index of graph operations. After it in paper [14] V.R. Kulli introduced the first hyper-Zagreb indices of a graph which was

$$HII_1(G) = \prod_{uv \in E(G)} (d_u + d_v)^2.$$

In the same paper [14] V.R. Kulli introduced the second hyper-Zagreb indices such as

$$HII_2(G) = \prod_{uv \in E(G)} (d_u \cdot d_v)^2.$$

In [15] Kulli et al. define the first and second generalized Zagreb indices which where

$$MZ_1^\alpha(G) = \prod_{uv \in E(G)} (d_u + d_v)^\alpha.$$

$$MZ_2^\alpha(G) = \prod_{uv \in E(G)} (d_u \cdot d_v)^\alpha.$$

Multiplicative sum and product connectivity indices defined in paper [16] such as

$$SCII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_u + d_v}},$$

$$PCII(G) = \prod_{uv \in E(G)} \frac{1}{\sqrt{d_u \cdot d_v}}.$$

The atom bound connectivity (ABC) index was firstly discussed in [17] then in [18] Multiplicative atomic bound connectivity index and Geometric arithmetic index [19] were defined, which were

$$ABCII(G) = \prod_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u \cdot d_v}},$$

$$GAI(G) = \prod_{uv \in E(G)} \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v} \right).$$

The general geometric arithmetic index was define in [20], which was

$$GA^\alpha II(G) = \prod_{uv \in E(G)} \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v} \right)^\alpha.$$

3. MAIN RESULTS

Calculations of degree based multiplicative topological indices of alcohol will be done in this section. Let G be the graph of alcohol structure, for alcohol structure we can rift $E(G) = E_1(G) \cup E_2(G) \cup E_3(G) \cup E_4(G)$, where

$$E_1(G) = \{e = uv \in E(G) \mid d_u = 4, d_v = 1\},$$

$$E_2(G) = \{e = uv \in E(G) \mid d_u = 4, d_v = 2\},$$

$$E_3(G) = \{e = uv \in E(G) \mid d_u = 2, d_v = 1\},$$

$$E_4(G) = \{e = uv \in E(G) \mid d_u = 4, d_v = 4\}.$$

It's easy for the reader to calculate that $|E_1(G)| = 2n + 1$, $|E_2(G)| = 1$, $|E_3(G)| = 1$ and $|E_4(G)| = n - 1$. Calculations will be same for all type of alcohols.

Theorem 1. First multiplicative Zagreb index for alcohol is

$$II_1^*(G) = 2^{3n-2} \times 3^2 \times 5^{2n+1}.$$

Proof. As we know that

$$\begin{aligned} II_1^*(G) &= \prod_{uv \in E(G)} (d_u + d_v), \\ &= \prod_{uv \in E_1(G)} (d_u + d_v) \times \prod_{uv \in E_2(G)} (d_u + d_v) \times \prod_{uv \in E_3(G)} (d_u + d_v) \times \prod_{uv \in E_4(G)} (d_u + d_v), \\ &= (4 + 1)^{|E_1(G)|} \times (4 + 2)^{|E_2(G)|} \times (2 + 1)^{|E_3(G)|} \times (4 + 4)^{|E_4(G)|}, \\ &= 5^{2n+1} \times 6 \times 3 \times 8^{n-1}, \\ &= 2^{3n-2} \times 3^2 \times 5^{2n+1}. \end{aligned}$$

Theorem 2. Second multiplicative Zagreb index for alcohol is

$$II_2(G) = 2^{8n+2}.$$

Proof. As we know that

$$\begin{aligned} II_2(G) &= \prod_{uv \in E(G)} (d_u \cdot d_v), \\ &= \prod_{uv \in E_1(G)} (d_u \cdot d_v) \times \prod_{uv \in E_2(G)} (d_u \cdot d_v) \times \prod_{uv \in E_3(G)} (d_u \cdot d_v) \times \prod_{uv \in E_4(G)} (d_u \cdot d_v), \\ &= (4)^{|E_1(G)|} \times (8)^{|E_2(G)|} \times (2)^{|E_3(G)|} \times (16)^{|E_4(G)|}, \\ &= 2^{4n+2} \times 2^3 \times 2 \times 2^{4n-4}, \\ &= 2^{8n+2}. \end{aligned}$$

Theorem 3. First multiplicative hyper Zagreb index for alcohol is

$$HII_1(G) = 2^{6n-4} \times 3^4 \times 5^{4n+2}.$$

Proof. As we know that

$$\begin{aligned} HII_1(G) &= \prod_{uv \in E(G)} (d_u + d_v)^2, \\ &= \prod_{uv \in E_1(G)} (d_u + d_v)^2 \times \prod_{uv \in E_2(G)} (d_u + d_v)^2 \times \prod_{uv \in E_3(G)} (d_u + d_v)^2 \times \prod_{uv \in E_4(G)} (d_u + d_v)^2, \\ &= (4 + 1)^{2|E_1(G)|} \times (4 + 2)^{2|E_2(G)|} \times (2 + 1)^{2|E_3(G)|} \times (4 + 4)^{2|E_4(G)|}, \\ &= 5^{4n+2} \times 6^2 \times 3^2 \times 8^{2n-2}, \\ &= 2^{6n-4} \times 3^4 \times 5^{4n+2}. \end{aligned}$$

Theorem 4. Second multiplicative hyper Zagreb index for alcohol is

$$HII_2(G) = 2^{16n+4}.$$

Proof. As we know that

$$\begin{aligned} HII_2(G) &= \prod_{uv \in E(G)} (d_u \cdot d_v)^2, \\ &= \prod_{uv \in E_1(G)} (d_u \cdot d_v)^2 \times \prod_{uv \in E_2(G)} (d_u \cdot d_v)^2 \times \prod_{uv \in E_3(G)} (d_u \cdot d_v)^2 \times \prod_{uv \in E_4(G)} (d_u \cdot d_v)^2, \\ &= (4)^{2|E_1(G)|} \times (8)^{2|E_2(G)|} \times (2)^{2|E_3(G)|} \times (16)^{2|E_4(G)|}, \\ &= 2^{8n+4} \times 2^6 \times 2^2 \times 2^{8n-8}, \\ &= 2^{16n+4}. \end{aligned}$$

Theorem 5. First multiplicative generalized Zagreb index for alcohol is

$$MZ_1^\alpha(G) = 2^{\alpha(3n-2)} \times 3^{2\alpha} \times 5^{\alpha(2n+1)}.$$

Proof. As we know that

$$\begin{aligned} MZ_1^\alpha(G) &= \prod_{uv \in E(G)} (d_u + d_v)^\alpha, \\ &= \prod_{uv \in E_1(G)} (d_u + d_v)^\alpha \times \prod_{uv \in E_2(G)} (d_u + d_v)^\alpha \times \prod_{uv \in E_3(G)} (d_u + d_v)^\alpha \times \prod_{uv \in E_4(G)} (d_u + d_v)^\alpha, \\ &= (4 + 1)^{\alpha|E_1(G)|} \times (4 + 2)^{\alpha|E_2(G)|} \times (2 + 1)^{\alpha|E_3(G)|} \times (4 + 4)^{\alpha|E_4(G)|}, \\ &= 5^{\alpha(2n+1)} \times 2^{2\alpha} \times 3^{2\alpha} \times 2^{3\alpha(n-1)}, \\ &= 2^{\alpha(3n-2)} \times 3^{2\alpha} \times 5^{\alpha(2n+1)}. \end{aligned}$$

Theorem 6. Second multiplicative generalized Zagreb index for alcohol is

$$MZ_2^\alpha(G) = 2^{\alpha(8n+2)}.$$

Proof. As we know that

$$\begin{aligned} MZ_2^\alpha(G) &= \prod_{uv \in E(G)} (d_u \cdot d_v)^\alpha, \\ &= \prod_{uv \in E_1(G)} (d_u \cdot d_v)^\alpha \times \prod_{uv \in E_2(G)} (d_u \cdot d_v)^\alpha \times \prod_{uv \in E_3(G)} (d_u \cdot d_v)^\alpha \times \prod_{uv \in E_4(G)} (d_u \cdot d_v)^\alpha, \\ &= (4)^\alpha |E_1(G)| \times (8)^\alpha |E_2(G)| \times (2)^\alpha |E_3(G)| \times (16)^\alpha |E_4(G)|, \\ &= 2^{\alpha(4n+2+4+4n-4)}, \\ &= 2^{\alpha(8n+2)}. \end{aligned}$$

Theorem 7. Multiplicative sum connectivity index for alcohol is

$$SCII(G) = \frac{1}{2^{\frac{3n-2}{2}}} \times \frac{1}{3} \times \frac{1}{5^{\frac{2n+1}{2}}}.$$

Proof. As we know that

$$\begin{aligned} SCII(G) &= \prod_{uv \in E(G)} \frac{1}{\sqrt{d_u+d_v}}, \\ &= \prod_{uv \in E_1(G)} \frac{1}{\sqrt{d_u+d_v}} \times \prod_{uv \in E_2(G)} \frac{1}{\sqrt{d_u+d_v}} \times \prod_{uv \in E_3(G)} \frac{1}{\sqrt{d_u+d_v}} \times \prod_{uv \in E_4(G)} \frac{1}{\sqrt{d_u+d_v}}, \\ &= \left(\frac{1}{\sqrt{5}}\right)^{|E_1(G)|} \times \left(\frac{1}{\sqrt{6}}\right)^{|E_2(G)|} \times \left(\frac{1}{\sqrt{3}}\right)^{|E_3(G)|} \times \left(\frac{1}{\sqrt{8}}\right)^{|E_4(G)|}, \\ &= \frac{1}{5^{2n+1}} \times \frac{1}{2^2 \cdot 3^2} \times \frac{1}{3^2} \times \frac{1}{2^{\frac{3n-3}{2}}}, \\ &= \frac{1}{2^{\frac{3n-2}{2}}} \times \frac{1}{3} \times \frac{1}{5^{\frac{2n+1}{2}}}. \end{aligned}$$

Theorem 8. Multiplicative product connectivity index for alcohol is

$$PCII(G) = \frac{1}{2^{4n+1}}.$$

Proof. As we know that

$$\begin{aligned} PCII(G) &= \prod_{uv \in E(G)} \frac{1}{\sqrt{d_u \cdot d_v}}, \\ &= \prod_{uv \in E_1(G)} \frac{1}{\sqrt{d_u \cdot d_v}} \times \prod_{uv \in E_2(G)} \frac{1}{\sqrt{d_u \cdot d_v}} \times \prod_{uv \in E_3(G)} \frac{1}{\sqrt{d_u \cdot d_v}} \times \prod_{uv \in E_4(G)} \frac{1}{\sqrt{d_u \cdot d_v}}, \\ &= \left(\frac{1}{\sqrt{4}}\right)^{|E_1(G)|} \times \left(\frac{1}{\sqrt{8}}\right)^{|E_2(G)|} \times \left(\frac{1}{\sqrt{2}}\right)^{|E_3(G)|} \times \left(\frac{1}{\sqrt{16}}\right)^{|E_4(G)|}, \\ &= \frac{1}{2^{2n+1}} \times \frac{1}{2^{\frac{3}{2}}} \times \frac{1}{2^{\frac{1}{2}}} \times \frac{1}{2^{2n-1}}, \\ &= \frac{1}{2^{4n+1}}. \end{aligned}$$

Theorem 9. Multiplicative atomic bound connectivity index for alcohol is

$$ABCII(G) = \frac{3^{3n}}{2^{\frac{7n+1}{2}}}.$$

Proof. As we know that

$$\begin{aligned} ABCII(G) &= \prod_{uv \in E(G)} \sqrt{\frac{d_u+d_v-2}{d_u \cdot d_v}}, \\ &= \prod_{uv \in E_1(G)} \sqrt{\frac{d_u+d_v-2}{d_u \cdot d_v}} \times \prod_{uv \in E_2(G)} \sqrt{\frac{d_u+d_v-2}{d_u \cdot d_v}} \times \prod_{uv \in E_3(G)} \sqrt{\frac{d_u+d_v-2}{d_u \cdot d_v}} \times \prod_{uv \in E_4(G)} \sqrt{\frac{d_u+d_v-2}{d_u \cdot d_v}}, \\ &= \left(\sqrt{\frac{3}{4}}\right)^{|E_1(G)|} \times \left(\sqrt{\frac{4}{8}}\right)^{|E_2(G)|} \times \left(\sqrt{\frac{1}{2}}\right)^{|E_3(G)|} \times \left(\sqrt{\frac{6}{16}}\right)^{|E_4(G)|}, \\ &= \frac{3^{\frac{2n+1}{2}}}{2^{2n+1}} \times \frac{1}{2^{\frac{1}{2}}} \times \frac{1}{2^{\frac{1}{2}}} \times \frac{2^{\frac{n-1}{2}} \cdot 3^{\frac{n-1}{2}}}{2^{2n-2}}, \\ &= \frac{3^{3n}}{2^{\frac{7n+1}{2}}}. \end{aligned}$$

Theorem 10. Multiplicative geometric arithmetic index for alcohol is

$$GAI(G) = \frac{2^{4n+5}}{3^2 \cdot 5^{2n+1}}.$$

Proof. As we know that

$$\begin{aligned} GAI(G) &= \prod_{uv \in E(G)} \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v} \right), \\ &= \prod_{uv \in E_1(G)} \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v} \right) \times \prod_{uv \in E_2(G)} \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v} \right) \times \prod_{uv \in E_3(G)} \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v} \right) \times \prod_{uv \in E_4(G)} \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v} \right), \\ &= \left(\frac{2\sqrt{4}}{4+1} \right)^{|E_1(G)|} \times \left(\frac{2\sqrt{4 \times 2}}{4+2} \right)^{|E_2(G)|} \times \left(\frac{2\sqrt{2}}{2+1} \right)^{|E_3(G)|} \times \left(\frac{2\sqrt{4 \times 4}}{4+4} \right)^{|E_4(G)|}, \\ &= \frac{2^{4n+2}}{5^{2n+1}} \times \frac{2^{\frac{5}{2}}}{2 \times 3} \times \frac{2^{\frac{3}{2}}}{3}, \\ &= \frac{2^{4n+5}}{3^2 \cdot 5^{2n+1}}. \end{aligned}$$

Theorem 11. Generalized multiplicative geometric arithmetic index for alcohol is

$$GA^\alpha II(G) = \frac{2^{\alpha(4n+5)}}{3^{2\alpha} \cdot 5^{\alpha(2n+1)}}.$$

Proof. As we know that

$$\begin{aligned} GA^\alpha II(G) &= \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v} \right)^\alpha, \\ &= \prod_{uv \in E_1(G)} \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v} \right)^\alpha \times \prod_{uv \in E_2(G)} \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v} \right)^\alpha \times \prod_{uv \in E_3(G)} \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v} \right)^\alpha \times \prod_{uv \in E_4(G)} \left(\frac{2\sqrt{d_u \cdot d_v}}{d_u + d_v} \right)^\alpha, \\ &= \left(\frac{2\sqrt{4}}{4+1} \right)^{\alpha|E_1(G)|} \times \left(\frac{2\sqrt{4 \times 2}}{4+2} \right)^{\alpha|E_2(G)|} \times \left(\frac{2\sqrt{2}}{2+1} \right)^{\alpha|E_3(G)|} \times \left(\frac{2\sqrt{4 \times 4}}{4+4} \right)^{\alpha|E_4(G)|}, \\ &= \frac{2^{\alpha(4n+2)}}{5^{\alpha(2n+1)}} \times \frac{2^{\alpha(\frac{5}{2})}}{2^\alpha \times 3^\alpha} \times \frac{2^{\alpha(\frac{3}{2})}}{3^\alpha}, \\ &= \frac{2^{\alpha(4n+5)}}{3^{2\alpha} \cdot 5^{\alpha(2n+1)}}. \end{aligned}$$

4. SUMMARY & CONCLUSION

In this paper, we have calculated some multiplicative degree based topological indices of a very important functional group alcohol. We have computed first multiplicative Zagreb index $II_1^*(G)$, second multiplicative Zagreb index $II_2(G)$, First multiplicative hyper Zagreb index $HII_1(G)$, Second multiplicative hyper Zagreb index $HII_2(G)$, First multiplicative α generalized Zagreb index $MZ_1^\alpha(G)$, Second multiplicative generalized Zagreb index $MZ_2^\alpha(G)$, Multiplicative sum connectivity index $SCII(G)$, Multiplicative product connectivity index $PCII(G)$, Multiplicative atomic bound connectivity index $ABCII(G)$, Multiplicative geometric arithmetic index $GAI(G)$ and Generalized multiplicative geometric arithmetic index $GA^\alpha II(G)$.

5. CONFLICT OF INTERESTS

We declare that there is no conflict of interests.

6. DATA AVAILABILTY

All the data is within the paper.

REFERENCES

- [1] I.Gutman "Multiplicative Zagreb indices of trees." Bull. Soc. Math. Banja Luka 18 (2011): 17-23.
- [2] A.Iranmanesh, M. A.Hosseinzadeh, and I. Gutman. "On multiplicative Zagreb indices of graphs." Iranian Journal of Mathematical Chemistry 3, no. 2 (2012): 145-154.
- [3] G.MODJTABA, and N.AZIMI. "Note on multiple Zagreb indices." Iranian Journal of Mathematical Chemistry 3, no. 2 (2012): 137-143.
- [4] N. Trinajstic, Chemical Graph Theory, CRC Press, Boca Raton, 1992.
- [5] A.Streitwieser, C.H.Heathcock, E.M.Kosower and P.J.Cornield 1992. Introduction to organic chemistry (Vol. 643). New York: Macmillan. 1992.
- [6] A.I.Vogel, A text-book of practical organic chemistry including qualitative organic analysis. Longmans Green And Co; London; New York; Toronto, 2013.
- [7] J. Soria and P. Tradacete, Best constants for the Hardy-Littlewood maximal operator on finite graphs, J. Math. Anal. Appl. 436 (2016), 661-682.
- [8] S.Akhter, M.Imran Computing the forgotten topological index of four operations on graphs. AKCE International Journal of Graphs and Combinatorics. 2017 Apr 1;14(1):70-79.
- [9] H.S.Ramane, V.M.Vinayak and G.Ivan, "General sum-connectivity index, general product-connectivity index, general Zagreb index and coindices of line graph of subdivision graphs." AKCE International Journal of Graphs and Combinatorics 14, no. 1 (2017): 92-100.
- [10] H.Narumi and M.Katayama, Simple topological index: A newly devised index characterizing the topological nature of structural isomers of saturated hydrocarbons, Memoirs of the Faculty of Engineering, Hokkaido University, 1984, 16(3), 209-214. (JIAPAC), 13(4), 335-341.
- [11] R.Kazemi, "The ratio and product of the multiplicative Zagreb indices." Iranian Journal of Mathematical Chemistry 8, no. 4 (2017): 377-390.
- [12] M.Eliasi, A.Iranmanesh, I.Gutman, Multiplicative versions of first Zagreb index, Match Communications in Mathematical and Computer Chemistry, 2012, 68(1), 217.
- [13] G. H.Shirdel, H.Rezapour, and A. M.Sayadi. "The hyper-Zagreb index of graph operations." Iranian Journal of Mathematical Chemistry 4, no. 2 (2013): 213-220.
- [14] V.R.Kulli, Multiplicative Hyper-Zagreb indices and coindices of graphs: computing these indecies of dome nanostructures. International Research Journal of Pure Algebra ISSN: 2248- 9037, 6(7)2016.
- [15] V.R.Kulli, B.Stone, S.Wang, B.Wei, Generalised multiplicative indices of polycyclic aromatic hydrocarbons and benzenoid systems, Zeitschrift fr Naturforschung A, 2017, 72(6),573-576.
- [16] V.R.Kulli, Multiplicative connectivity indices of TUC4C8 [m, n] and TUC4 [m, n] nanotubes, Journal of Computer and Mathematical Sciences, 2016, 7(11), 599-605.Scand 90: 145-156.

- [17] Fronimos I., E. Estrada, L. Torres, L. Rodriguez, I. Gutman An atombond connectivity index: Modelling the enthalpy of formation of alkanes. *Indian J. Chem.*, 37A (1998), pp. 849-855.
- [18] S.Shigehalli, R.Kanabur, Computation of New Degree-Based Topological Indices of Graphene, Volume 2016, Article ID 4341919, 6 pages 2016.
- [19] J.Palacios, "Some remarks on the arithmetic-geometric index." *Iranian Journal of Mathematical Chemistry* 9, no. 2 (2018): 113-120. *Development* 74(1), 70-75.
- [20] E.Mehdi, and A.Iranmanesh. "On ordinary generalized geometricarithmetic index." *Applied Mathematics Letters* 24, no. 4 (2011): 582-587. 14(2), 125-133.