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Analysis of Soliton Interaction with Higher Order Effects in Erbium-Doped Fiber

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ABSTRACT

We observe soliton propagation in an erbium-doped fiber .The exact soliton solutions are accomplished by Inhomogeneous Hirota Maxwell Bloch equation with variable nonlinear and dispersion parameter. For this system, we are introducing higher order effects like third order dispersion, Stimulated Raman Scattering (SRS) and self-steepening effects. By using the corresponding Lax pair for this equation is gathered through the AKNS technology. we have obtained multi-soliton solutions by using Lax pair and Darboux transformation method. In order to study the impact of soliton dynamics an inhomogeneous erbium doped fiber. As one of the special cases, pulse compression is discussed in detail and stability of the soliton is considered.

Keywords: Femtosecond soliton, Hirota Maxwell Bloch equation, EDFA, Lax pair, AKNS technology, Darboux transformation.

1. INTRODUCTION

The nonlinear Schrodinger equation (NLSE) is a fundamental model to describe various nonlinear physical phenomena in the field of nonlinear science such as optical solitons in optical fibers. [1] Optical solitons are contemplate to be the most important achievement on the road of communication technology. In order to make the optical soliton communication system more effective, competitive and economical, the attenuation in optical fibers should be avoided. This is because optical solitons are caused as a result of perfect balance between the group velocity dispersion (GVD) and the nonlinear effect, which are considered to be the major problems in fiber media. The GVD leads to temporal broadening of the optical pulse, due to the frequency dependence on the index of refraction. When input power increases refractive index becomes intensity dependent factor which leads to phase modulates itself is called Self Phase Modulation (SPM).The SPM produces Spectral broadening in optical pulses [2].

The generalized inhomogeneous nonlinear Schrodinger equation (GINLSE) model is essential and universal models of modern nonlinear science. Anyhow, in a real fiber, the core medium is not homogeneous [3]. We have encountered dispersion and nonlinearity separately. We have seen that for a linear dispersive transmission line a newly created pulse spreads out and disperses as it propagates. On the contrary, for a nonlinear non-dispersive transmission line, the profile of an initial pulse deforms and its wave front tends to become abrupt. In the following, we shall see that in a nonlinear dispersive transmission line the dispersion can balance the effects of nonlinearity, leading to a pulse-like wave that is, a solitary wave or a soliton which can propagate with constant velocity and profile. The most striking aspects of soliton behavior were seen by Scott Russell in a water tank which he described as being "a foot wide, eight or nine inches deep and twenty of thirty feet long". Following Russell's observations, a series of experiments were performed by Hammack and Segur (1974) in a big wave tank. Since then, simple water-tank experiments, which allow one to illustrate the important features of solitons in shallow water, have been described by Bettini et al. (1983) and alsen et al. (1984). Hasegawa and Tappert (1973) were the first to model the propagation of a guided mode in a perfect nonlinear monomode fiber by the NLS equation. They found that the optical pulse is an envelope or bright soliton and they predicted the stationary transmission of the pulse in the anomalous dispersion (ß2<O) regime. This prediction was then successfully verified by the experiments of Mollenauer et al. (1980).In Normal dispersion (ß2>O) regime supports Dark soliton. Those type of solitons are used to pulse compressing techniques [7].

Soliton interactions are perhaps the most fascinating feature of soliton phenomena. There are two categories: coherent and incoherent interactions, coherent interactions occur when the nonlinear medium can respond to interference effects that take place when the beams overlap. They occur for all nonlinearities with an instantaneous time response. Materials with a long response to interference between the overlapping beams, if the relative phase between the beams is stationary for a time much longer than retarded time. The soliton then exert attractive or repulsive forces on each others, depending on their relative phase. Incoherent interaction occur, when the relative phase between the (soliton) beams varies much faster than retarded time [8].

In real application to enhance information capacity, it is necessary to decrease the pulse width for increasing the bit rate. However, when the optical pulses become shorter, the NLS-typed equations not applicable to represent the pulse propagation in optical fibers. In this case, higher- order terms such as the third order dispersion (TOD) and simulated Raman scattering (SRS) effects become more important and must be taken into consideration. Thus, higher order nonlinear Schrodinger equation (HNLS) is used to describe the femtosecond soliton propagation in an ideal fiber. When practically speaking, the dispersion, nonlinearity, gain and loss are normally varied with the propagation distance and hence the propagation of femtosecond pulses is represent by an inhomogeneous higher order NLS (IHNLS) equation [2].

To rectify the losses in optical fibers, the rare-earth elements such as erbium atoms are doped with the core medium of the fiber. At particular wavelength this doping makes the fiber becomes a transparent medium to be both the silica and erbium atoms have an impact on soliton wave propagation. The optical pulse propagation in the erbium-doped fiber is constrained by the nonlinear Schrodinger–Maxwell–Bloch (NLS–MB) equations and the resultant solitons are called self-induced transparency (SIT) or NLS–MB solitons. The advantage of erbium doped fiber is commercially available in c band and L band, Insensitive to light polarization state, low noise figure, bit rate transparency, Immunity to cross talk among WDM channels[7] .Much earlier in 1967, McCall and Hahn explained this special kind of soliton known as the SIT soliton in a two-level resonant atomic medium. Nakazawa et al experimentally observed the coexistence of NLS solitons and SIT solitons in erbium-doped resonant fibers, and the relationship between soliton period and absorption length which characterizes the propagation property of SIT solitons has been discussed in[16].In recent years, terrific progress has been made in the development of broadband erbium-doped fiber amplifiers (EDFA), which form the backbone of high- capacity optical communication system for long-haul communication system [17].

2. THEORETICAL POINT

In this paper, we have to prospect the Inhomogeneous Hirota Maxwell Bloch model that represents Femtosecond soliton propagation in an erbium-doped fiber media in practical case. This system is known as the dispersion-nonlinearity managed soliton system to differentiate it from the DMS systems in which only the dispersion parameter is considered as inhomogeneous.

$$iq_z = -D_2(z)q_{tt} - 2R(z)|q|^2 q + iD_3(z)q_{ttt}$$
$$+ i\alpha(z)|q|^2 q_t + iG(z)q + \langle p \rangle$$

$$P_t = 2\eta f(z)q - 2i\omega p$$

$$\eta_t = -f(z)(qp^* + q^* p) \qquad (1)$$

Where q(z,t) is the complex envelope of the field. p(z,t) is the measure of the polarization of the resonant medium and η(z,t) denotes the extent of population inversion D2(z) represents the group velocity dispersion(GVD),D3(z) represents the third order dispersion, R(z) nonlinearity parameter, G(z) corresponds to gain or loss and f(z) is a parameter describing the interaction between the propagating field and energy levels of erbium atoms.<…> represents the averaging function over entire frequency range .For example,

$$\langle p(z,t) \rangle = \int_{-\infty}^{\infty} p(\omega t) g(\omega) d\omega \qquad (2)$$

Such that

$$\int_{-\infty}^{\infty} g(\omega) d\omega = 1$$

$g(\omega)$ being the distribution function which represents the uncertainty in the energy level of the resonant atoms.

3. LAX PAIR

On a method for constructing the Lax pairs for nonlinear integrable equations. Lax pairs have been constructed by many authors [8,9].In this work ,we alter their lax pairs so as to be a suitable for the variable coefficient Hirota-Maxwell Bloch equations as follows:

$$\varphi_t = U\varphi, \quad \varphi_z = V\varphi \qquad (3)$$

Where $\varphi = (\varphi_1, \varphi_2)^T$ and U and V are given by

$$U = \lambda J + P,$$

$$J = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad P = \sqrt{\frac{R(z)}{D_2(z)}} \begin{bmatrix} 0 & q \\ q^* & 0 \end{bmatrix}, \qquad (4)$$

$$v = \begin{bmatrix} A & B \\ C & -A \end{bmatrix} \qquad (5)$$

Where

$$A = 4D_3(z)\lambda^3 + 2iD_2(z)\lambda^2 + 2\frac{R(z)}{D_2(z)}D_3(z)|q|^2\lambda + iR(z)|q|^2 + \frac{R(z)}{D_2(z)}D_3(z)(q^*q_t - qq_t^*),$$

$$B = f(z)[4D_3(z)q\lambda^2 + 2(D_3(z)q_t + iD_2(z)q)\lambda + D_3(z)(q_{tt} + 2\frac{f^2(z)}{q|q|^2} + iD_2(z)q_t + i\left\langle\frac{-p}{\lambda - i\omega}\right\rangle]$$

$$C = f(z)[-4D_3(z)\bar{q}\lambda^2 + 2(D_3(z)\bar{q}_t - iD_2(z)q^*)\lambda - D_3(z)(q_n^* + 2f^2(z)q^*|q|^2) + iD_2(z)q_t^* + i\left\langle\frac{-p}{\lambda - i\omega}\right\rangle]$$

With the conditions

$$G(z) = \frac{1}{2}\frac{RD_{2z} - D_2R_z}{RD_2} \text{ and } f(z) = \left(\frac{R(z)}{D_2(z)}\right)^{\frac{1}{2}}$$

(6)

4. MULTI SOLITON SOLUTIONS

Among various methods, the Darboux transformation has been proved to be an efficient technique to find the soliton solution for NLS equations. we have implement this method to arrive the multi- soliton solution based on the obtained Lax pair as described below multi-soliton solutions are obtained via Darboux transformation (DT). The Darboux transformation method can also applied to a NLS equation with variable coefficient to derive a series of analytical solutions including the multi soliton solutions from an initial solution. For this purpose, we define the pseudo -potentials as[5]

$$\varphi = D\varphi = (\lambda I - S)\varphi \quad (7)$$

Where, D is called Darboux matrix.

$$S = H\Delta H^{-1} \quad (8)$$

Where H is a non singular matrix

$$H = \begin{bmatrix} \phi_1 & \bar{\phi}_2 \\ \phi_2 & \bar{\phi}_1 \end{bmatrix}$$

With

$$P_1 = \sqrt{\frac{R(z)}{D_2(z)}}\begin{bmatrix} 0 & q_1 \\ -\bar{q}_1 & 0 \end{bmatrix}$$

Thus we obtain the Darboux transformation eq(1) in the form

$$P_1 = P + SJ - JS \quad (9)$$

$$\Delta = \begin{bmatrix} \lambda & 0 \\ 0 & -\lambda \end{bmatrix} \quad (10)$$

Equation (9) substitute in equation (8), then

We get the value of S matrix.

$$S_{kl} = \frac{(\lambda_1 + \overline{\lambda_1})\varphi_k \varphi_l}{\Delta} \quad (11)$$

By substituting S_{kl} values, we find general solution

$$q_n = q + 2\sqrt{\frac{D}{R}} \frac{\Sigma(\lambda_m + \lambda_m^*)\varphi_{1,m}\lambda_m \varphi^*{}_{2,m}\lambda_m}{A_m} \quad (12)$$

$$A_m = \left|\varphi_{1,m}(\lambda_m)\right|^2 + \left|\varphi_{2,m}(\lambda_m)\right|^2 \quad (13)$$

Where m = 1, 2, . . , n, k = 1, 2, and $(\varphi_{1,1}(\lambda_1), \varphi_{2,1}(\lambda_1))^T$ is the Eigen function corresponding to λ_1 for q.

One soliton solution for the inhomogeneous Hirota system with MB part

Where

$$A = 2\alpha t + 8\int_0^z D_3(z)\alpha^3 dz - 24\int_0^z D_3(z)\alpha\beta^2 dz - 8\int_0^z \alpha D_2(z)\beta dz + 2\left\langle \frac{\beta - \omega}{\alpha^2 + (\beta - \omega)^2} \right\rangle \int_0^z f(z)dz + \chi_0 \quad (15)$$

$$B = 2\beta t + 24\int_0^z D_3(z)\alpha^2 \beta dz - 8\int_0^z D_3(z)\beta dz - 4\int_0^z D_2(z)(\alpha^2 - \beta^2)dz + 2\left\langle \frac{\alpha}{\alpha^2 + (\beta - \omega)^2} \right\rangle \int_0^z f(z)dz + \phi_0 \quad (16)$$

Where and are integration phase constants, which corresponding to initial position and initial phase respectively. The real part of the spectral parameter is corresponds to amplitude of the soliton and imaginary part related to velocity of the soliton pulses. Using the one-soliton solution as the seed solution , we can generate the two-soliton solution. Thus in recursion, one can generate up to n-soliton solution. Here, we present only the two-soliton solution in explicit forms to analyze the system (1). For the choice of n = 2 in Equation (11), we arrived two-soliton solution as given below

$$q_2 = 2\sqrt{\frac{D_3(z)}{R(z)}}[\alpha_1 \exp(iA_1)\sec h(B_1) + 2\alpha_2 \left(\frac{G}{F}\right)] \quad (17)$$

Where G and F are as below:

$$G = -e^{-2(iB_2 + A_1 + A_2)}((e^{2(iB_2 + A_1)} - (e^{2iB_2} + e^{2(iB_1 + A_1 + A_2)}) \sec h(2A_1)\alpha_1 + e^{2(iB_2 + A_1)}(\alpha_2 + i(\beta_1 - \beta_2))$$
$$(e^{2iB_1} \sec h(2A_1)\alpha_1 + e^{2(iB_2 + A_2)}(-\alpha_2 + i(\beta_1 - \beta_2)) + \alpha_1 \tanh(2A_1))) \quad (18)$$

$$F = 2(Cosh(2A_2)\alpha_1^2 + (-2\cos 2(B_1 - B_2) + \cosh(2(A_1 - A_2) - \cosh(2(A_1 + A_2))$$
$$\sec h(2A_1)\alpha_1\alpha_2 + \cosh(2A_2)) \quad (19)$$

The above two soliton solution shows that the pulse width, amplification, pulse compression and group velocity are related to the variable-coefficient parameters in soliton control systems, which occurs that bountiful femtosecond soliton structures can be obtained by adjusting these variable parameters.

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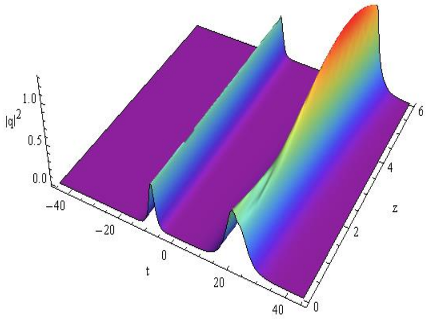
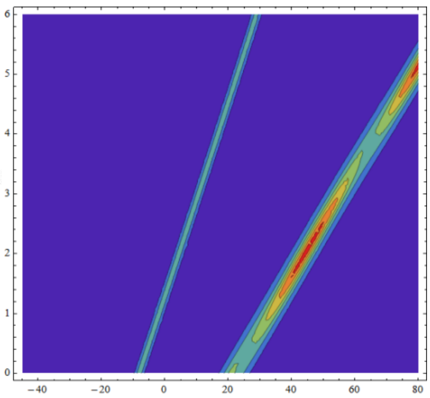
5. PULSE WIDTH MANAGEMENT

The optimal soliton pulse amplification can be efficiently achieved on employing soliton pulse width management. This is the one of essential need for designing fiber optic communication. Let us consider the pulse width management of soliton pulse in an optical fiber and for this purpose, we assume that the GVD and the nonlinearity functions are distributed in the form,

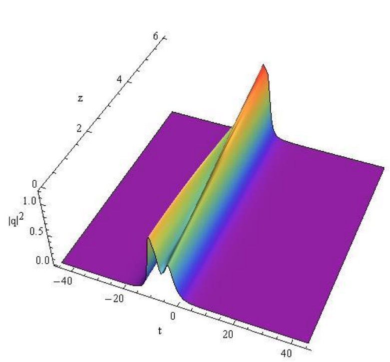
$$D_2(z) = D_3(z) = R(z) = e^{Kz} \quad (20)$$

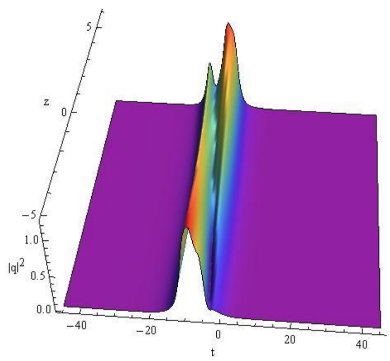
At present Soliton pulse compression techniques are more interest because of their simplicity dominance over other techniques. In this manuscript, we consider a system with both nonlinear and dispersion parameter treated as a inhomogeneous. By manipulating various physical parameter such as amplitude and velocity of pulse, we improve the soliton stability during long distance communication channels. We have plotted the femtosecond solitons for the above conditions by controlling the parameter K. If we taken as a low value of K=0.07, Dispersion managed solitons are formed as shown in Figure 2(a).After propagation pulse width controlled which is illustrated in the Figure2(b).The effect of erbium doped ions produces phase shift occurs in soliton pulses which makes interaction between two solitons. At initial position two solitons are propagating itself .In figure 2(c) and 2(d) high amplitude signal is considered as a energy reservoir (pump signal) and another signal act as a information signal. During propagation the energy is transferred from pump signal and the information signal gets amplified and stably transmitted over long distance communication.





(a) (b)





(c) (d)

Figure2.(a)The evolution of two soliton solution without erbium-doped fiber In this case(20), the corresponding variable parameters are adopted by $k=0.07; \omega_1=7; \omega_2=10; \alpha_1=-0.23; \beta_1=0.27; \alpha_2=0.25; \beta_2=-0.45$.(b) contour plot of (a).(c)The evolution of two soliton solution with interaction In this case(20), the corresponding variable parameters are adopted by $k=0.07; \omega_1=17; \omega_2=10; \alpha_1=0.23; \beta_1=-0.25; \alpha_2=0.27; \beta_2=-0.25$.(d) contour plot of (c)

6. CONCLUSION

In this manuscript, we have studied the inhomogeneous system of the Hirota-Maxwell Bloch equation .Which describes the ultra-short pulse propagation in an erbium-doped fiber with variable dispersion, nonlinearity and gain/loss parameters. Lax pair is constructed for this equation through the AKNS technology and two soliton solutions are obtained by using Darboux transformation method. Using obtained soliton solution, pulse width management is examined in detail and stability of the pulse is discussed. In soliton interaction to achieve amplification without using any external amplifier. These results has potential applications in the field of soliton based communication system.

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