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Analysis of Parameters Estimation for Three Independent Competing Risks *Under Type-II* Censoring

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ABSTRACT

This paper mainly aims to estimate the unknown parameters included in three independent competing risks in the presence of "complete observations", "incomplete observations" and Type-II censoring. Analyzing data is established from some of Weibull models with consideration mainly that there are three independent causes of failure. The case when the competing risks (three causes) have Weibull distribution with two parameters, exponentiated Weibull distribution and Rayleigh distribution is considered. The maximum likelihood estimators of the different parameters and their relative risks are obtained. Properties of the estimated values have been studied through a simulation study.

Keywords: Competing Risks, Complete observations, Incomplete observations, Censored data, Maximum likelihood estimators and Relative risk.

1. INTRODUCTION

In many applications in survival analysis it is quite common to have several possible risks (causes) of failure present at the same time. The actual cause of failure of an item may be one, and only one, of these risks. Hence these risks are said to compete for the life of the item. Models for lifetime in the presence of such competing risks in statistical literature are known as the competing risk models. [see Smith *et al* (2003)]

Competing risk models have been studied by several authors using parametric and non-parametric setup. The parametric setup is performed assuming that the competing risks follow different lifetime distributions such as exponential, gamma, Weibull, and generalized exponential distributions, see for example Berkson and Elveback (1960), Cox (1959), David and Moeschberger (1978), Park (2005), Kundu and Sarhan (2006) and Sarhan (2007). The non-parametric setup does not consider a specific lifetime distribution. The analysis of the non-parametric version of this model has been investigated by several authors such as Kaplan and Meier (1958).

In many applications, in analyzing competing risk models, it is assumed that each observation has a failure time and an indicator denoting the cause of failure. It is usually assumed, in either parametric or non-parametric models, that both the failure times and the causes of failure are observed. This situation is referred to as the case of complete observation. However, in certain situations, the determination of the cause of failure may be expensive and requires time, very difficult or impossible to observe. Thus it might happen that the failure time of that item is observed but the corresponding cause of failure is not observed [see Alwasel (2009)]. Kundu and Basu (2000) considered the following two types of observations:

a) The item has failed due to a certain cause of failure, and both its time of failure and the cause are known (complete observation).

b) The item has failed, and its time of failure has been known, but the cause of failure is unknown (incomplete observation).

Also, they assumed that every observation in the sample can be monitored until failure. That is, there is no censoring. But in most applications, some observations may be alive at the end of the project period; that is, the data are censored; see for example David and Moeschberger (1978). In addition to the above types of observations, Sarhan (2007) considered the following third type:

c) The item was still working at the end of the project period. Naturally, he referred to it as censored observations.

Censoring is unavoidable in life testing and survival studies because the experimenter is unable to obtain complete information on lifetime for all observations. For example, patients in a clinical trial may withdraw from the study, or the study may have to be terminated at a pre-fixed time point. In industrial experiments, units may break accidentally. The two most common censoring schemes are termed *Type-I* and *Type-II* censoring.

***Type-I* censoring:**

Occurs if an experiment has a set number of observations and the experiment stopped at a predetermined time.

Type-II censoring:

Occurs if an experiment has a set number of observations and the experiment stopped when a predetermined number are observed to have failed.

El-Kelany (2012) presented competing risk models in survival analysis with some Wiebull models in the case of two independent causes of failures. She considered two cases. The first case, when the first cause has a Weibull (*W*) distribution with two parameters and the second cause has an exponentiated Weibull (*EW*) distribution under *Type-I* and *Type-II* censoring, respectively. The second case, when the first cause has a (*W*) distribution with two parameters and the second cause has Rayleigh (*R*) distribution under *Type-I* and *Type-II* censoring, respectively.

El-Kelany (2015) presented three independent competing risk model in survival analysis with some Wiebull models in the case of *Type-I* censoring. She considered the case, when the first cause has a *W* distribution with two parameters and the second cause has an *EW* distribution and the third cause has *R* distribution under *Type-I* censoring.

This paper is concerned with a case of three independent causes of failure under *Type-II* censoring . It considered the case when the competing risks (three causes) have *W* distribution with two parameters, *EW* distribution and *R* distribution, respectively in the presence of complete observations, incomplete observations under *Type-II* censoring, since it is the mode of censoring most common in practice.

The *MLEs* of different parameters with different sampling schemes and their properties are studied under these assumptions. A simulation study is conducted for studying the properties of the estimators for the unknown parameters.

The rest of the paper is organized as follows: Notations which are needed for describing the model are presented in Section *2.* Section 3 is concerned with the suggested model. The *MLEs* of the unknown parameters under *Type-II* censoring are considered in Section 4.The relative risk rates are obtained in Section *5*. A simulation study is conducted in Section *6* and concluding remarks are presented in Section *7*.

2. NOTATIONS

n	the sample size.
k	number of causes.
m	Number of censored data in *Type-II* censoring.
f. (.)	the probability density function, (*p.d.f*).
F.(.)	the cumulative distribution function, (*c.d.f*).
S. (.)	the survival function, (*s.f*).
h. (.)	the hazard function, (*h.f*).
$\delta_i = j$	indicator variable means the observation i has failed at time T_i due to cause j, j=1,2 and 3
$\delta_i =*$	means the cause of failure of observation i is unknown.
I(.)	indicator function of the observation (.)
MLEs	The maximum likelihood estimators.

3. THE SUGGESTRD MODEL

The model suggested in the present study is a three independent competing risks model. The case considered here when the first cause has *W* distribution with two parameters, the second cause has *EW* distribution and the third cause has *R* distribution. In sub-section 3.1 a brief summary to these distributions is introduced and in sub-section 3.2 the model assumptions are illustrated.

3.1 SOME WEIBULL MODELS

3.1.1 WEIBULL DISTRIBUTION

Weibull (*W*) distribution is often used in the field of life data analysis due to its flexibility. It is commonly used to model systems with monotone failure rates. This paper is concerned with the two parameters *W* distribution. It can be used to analyze lifetime data because it has a variety models of life behaviors. The *p.d.f* of the two parameters *W* distribution is:

$$f(t;\alpha,\lambda)=\alpha\lambda t^{\alpha-1}\, e^{-\lambda t^{\alpha}} \quad ;t>0,\alpha>0,\lambda>0, \tag{1}$$

where α is a shape parameter, λ is a scale parameter. If α < 1, the failure rate decreases over time t. If α =1, the failure rate is constant over time t. If α >1, the failure rate increases over time t. The corresponding *c.d.f*, *s.f* and *h.f* (failure rate) respectively are:

$$F(t)=1-e^{-\lambda t^{\alpha}} \tag{2}$$

$$S(t)=e^{-\lambda t^{\alpha}} \tag{3}$$

$$h(t)=\alpha\lambda t^{\alpha-1} \tag{4}$$

3.1.2 THE EXPONENTIATED DISTRIBUTION

Mudholkar and Srivastava (1993) introduced *EW* distribution and later Mudholkar*et al.* (1995) made further studies on this distribution. Its properties have been studied in more detail by Mudholkar and Hutson (1996) and Nassar and Eissa (2003). The *EW* family is an extension of *W* distribution obtained by adding an extra shape parameter. The importance of this distribution lies in its ability to model monotone as well as non-monotone failure rates which are quite common in reliability and some biological studies.

The *p.d.f* of *EW* distribution is given by:

$$f(t,\alpha,\theta,\sigma) = \frac{\alpha\,\theta}{\sigma}\left(\frac{t}{\sigma}\right)^{\alpha-1} e^{-\left(\frac{t}{\sigma}\right)^{\alpha}}\left(1-e^{-\left(\frac{t}{\sigma}\right)^{\alpha}}\right)^{\theta-1} \tag{5}$$

where $\alpha>0$, $\theta>0$ are shape parameters and $\sigma>0$ is a scale parameter. As special cases, the *W* distribution when $\theta=1$ and the exponential distribution when $\alpha=1$ and $\theta=1$. The *c.d.f, s.f* and *h.f* respectively are given by:

$$F(t) = \left(1-e^{-\left(\frac{t}{\sigma}\right)^{\alpha}}\right)^{\theta} \tag{6}$$

$$S(t) = 1-\left(1-e^{-\left(\frac{t}{\sigma}\right)^{\alpha}}\right)^{\theta} \tag{7}$$

$$h(t) = \frac{\alpha\,\theta\left(\frac{t}{\sigma}\right)^{\alpha-1} e^{-\left(\frac{t}{\sigma}\right)^{\alpha}}\left(1-e^{-\left(\frac{t}{\sigma}\right)^{\alpha}}\right)^{\theta-1}}{\sigma\left(1-\left(1-e^{-\left(\frac{t}{\sigma}\right)^{\alpha}}\right)^{\theta}\right)} \tag{8}$$

The great flexibility of this model in fitting survival data can be depicted from its hazard function, which can be:

- Monotonically decreasing if $\alpha\le 1$ and $\alpha\,\theta\le 1$.
- Monotonically increasing if $\alpha\ge 1$ and $\alpha\,\theta\ge 1$.
- Bathtub shape if $\alpha>1$ and $\alpha\,\theta<1$.
- Unimodal if $\alpha<1$ and $\alpha\,\theta>1$ [see Edwin *et al* (2006)].

3.1.3 RAYLEIGH DISTRIBUTION

Rayleigh distribution is a special case of *W* distribution, that is, when $\alpha=2$ and $\lambda=\frac{1}{2\sigma^2}$. In this study, we are concerned with *R* distribution because it has a linearly increasing rate, so, it is appropriate for components which might not have manufacturing defects, but age rapidly with time. The *p.d.f* of the *R* distribution is:

$$f(t;\sigma)=\frac{t}{\sigma^2}e^{-\frac{t^2}{2\sigma^2}} \quad ;\sigma>0,t>0 \tag{9}$$

The corresponding *c.d.f, s.f* and *h.f* respectively are:

$$F(t)=1-e^{-\frac{t^2}{2\sigma^2}} \tag{10}$$

$$S(t)=e^{-\frac{t^2}{2\sigma^2}} \tag{11}$$

$$h(t)=\frac{t}{\sigma^2} \tag{12}$$

3-2 MODEL ASSUMPATIONS

1- The observation is exposed to k =3 independent competing risks of failure.

2- One and only one of these risks actually claim the life of the observation and it is called the cause of failure.

3- It is assumed that T_1, T_2, *and* T_3 are positive continuous random variables denoting the lifetime (time to failure) under the k risks.

4- The random latent failure times (T_{i1} , T_{i2} and T_{i3}); $i = 1,2,\ldots,n$ are independent random variables for all $i = 1, 2, \ldots, n$; hence $F(t) = F_1(t)\times F_2(t)\times F_3(t)$, [see Miyakawa (1984)].

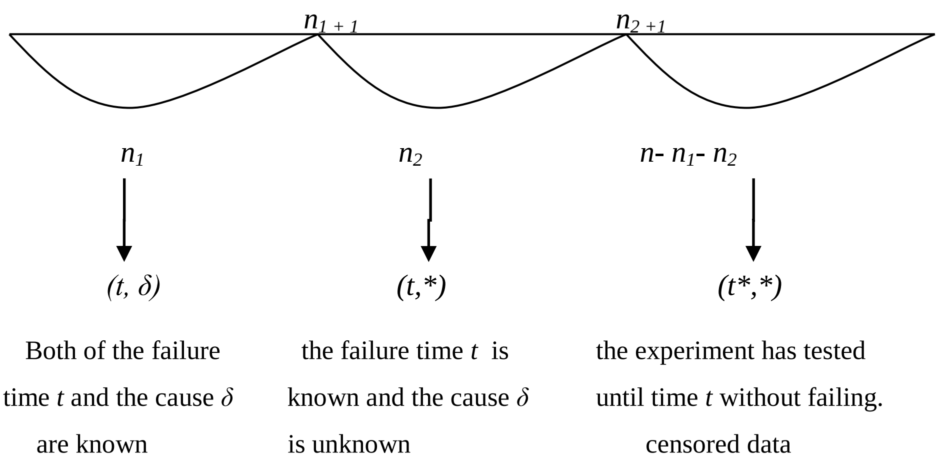
5- The latent failure time is T_i=min $[T_{i1},T_{i2}, T_{i3}]$

6- The t_{i1} denotes the failure time according to the first cause, which follows the W distribution defined in equation (1), t_{i2} denotes the failure time according to the second cause which follows the EW distribution defined in equation (5), t_{i3} denotes the failure time according to the third cause which follows the R distribution defined in equation (9) and m denotes the number of censored data in *Type-II* censoring

7- The probability that $[t_{i1}=, t_{i2}=, t_{i3}] = 0$

8- In the first (n_1) observations, the failure times and also causes of failure are observed. Whereas for the following (n_2) observations the failure times only are observed and not the causes of failure, that is the cause of failure is unknown. In the remaining ($n - n_1 - n_2$) observations, the experiment still alive at the end the project periods. Namely, we observe the following data:

$(t_1,\delta_1),(t_2,\delta_2),\ldots\ldots,(t_{n_1},\delta_{n_1}),(t_{n_1+1},*),\ldots\ldots,(t_{n_2},*),\ and\ (t^*_{n_2+1},*),\ldots\ldots\ldots,(t^*_n,*)$. Where:



We denote this set by Ω which can be categorized as a union of three disjoint classes' Ω_1, Ω_2 and Ω_3. Where Ω_1 represents the set of data when the cause of experiment failure is known, while Ω_2 denotes the set of observations when the cause of experiment failure is unknown and Ω_3 denotes the set of censored observations. Further, the set Ω_1 can be divided into three disjoint subsets of observations Ω_{11}, Ω_{12} and Ω_{13}, where Ω_{1j} represents the set of all observations when the failure of the experiment is due to the cause *j*, *j*= 1, 2 and 3. It is also assumed that $|\Omega_i|=r_i$, $|\Omega_{1j}|=r_{1j}$. Namely, $|\Omega_1|=r_1=(r_{11}+r_{12}+r_{13})$, $|\Omega_2|=r_2=(n_2)$ and $|\Omega_3|=r_3=(n-n_1-n_2)$.

4. THE MAXIMUM LIKELIHOOD ESTIMATORS

According to the assumptions which mentioned in subsection (3.2), for *Type-II* censoring, it is assumed that the experiment stops at a fixed number of failure, m, then the likelihood function of the observed data mentioned in equation (13), for the general case, takes the following form:

$$L=\prod_{i=1}^{n_1}\begin{pmatrix} (f_1(t_i)S_2(t_i)\ S_3(t_i))^{I(\delta_i=1)}(f_2(t_i)\ S_1(t_i)\ S_3(t_i))^{I(\delta_i=2)} \\ (f_3(t_i)\ S_1(t_i)\ S_2(t_i))^{I(\delta_i=3)} \end{pmatrix} \times \prod_{i=n_1+1}^{n_2}(d\,F(t_i)) \times \prod_{i=n_2+1}^{n}(S(t_i)) \,, \tag{13}$$

Where $d\,F(t_i)$ is the derivative of $F(t)$, [see Miyakawa (1984)]. Hence:

$$L=\prod_{i=1}^{n_1}\begin{pmatrix} (f_1(t_i)S_2(t_i)\ S_3(t_i))^{I(\delta_i=1)}(f_2(t_i)\ S_1(t_i)\ S_3(t_i))^{I(\delta_i=2)} \\ (f_3(t_i)\ S_1(t_i)\ S_2(t_i))^{I(\delta_i=3)} \end{pmatrix}$$
$$\times \prod_{i=n_1+1}^{n_2}(f_1(t_i)\ S_2(t_i)S_3(t_i)+f_2(t_i)\ S_1(t_i)S_3(t_i)+f_3(t_i)\ S_1(t_i)S_2(t_i))^{I(\delta_i=*)}$$
$$\times \prod_{i=n_2+1}^{n}(S(\tau)) \tag{14}$$

Where:

$$L=\prod_{i=1}^{n_1}\left\{\left[(\alpha\,\lambda\,t_i^{\alpha-1}\ e^{-\lambda\,t_i^{\alpha}})\left(e^{-\frac{t_i^{\alpha}}{2\sigma^2}}\right)\left(1-\left(1-\ e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)^{\theta}\right)\right]^{I(\delta_i=1)}\right.$$
$$*\left[\left(\frac{t_i}{\sigma^2}\ e^{-\frac{t_i^{\alpha}}{2\sigma^2}}\right)(e^{-\lambda\,t_i^{\alpha}})\left(1-\left(1-\ e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)^{\theta}\right)\right]^{I(\delta_i=2)}$$
$$\left.*\begin{bmatrix}\left(\frac{\alpha\,\theta}{\sigma}\left(\frac{t_i}{\sigma}\right)^{\alpha-1}\ e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)\left(1-e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)^{\theta-1} * \\ (e^{-\lambda\,t_i^{\alpha}})\left(e^{-\frac{t_i^{\alpha}}{2\sigma^2}}\right)\end{bmatrix}^{I(\delta_i=3)}\right\}$$
$$*\prod_{n_1+1}^{n_2}\left\{\left[\left(e^{-\lambda\,t_i^{\alpha}}\ e^{-\frac{t_i^{\alpha}}{2\sigma^2}}\right)(\alpha\,\lambda\,t_i^{\alpha-1})\left(1-\left(1-\ e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)^{\theta}\right)+\frac{t_i}{\sigma^2}\left(1-\left(1-\ e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)^{\theta}\right)\right.\right.$$
$$\left.\left.+\left(\frac{\alpha\,\theta}{\sigma}\left(\frac{t_i}{\sigma}\right)^{\alpha-1}\ e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)\left(1-\ e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)^{\theta-1}\right]^{I(\delta_i=*)}\right\}$$
$$*\prod_{n_2+1}^{n}\left\{\left(e^{-\lambda\,\tau_i^{\alpha}}\,e^{-\frac{\tau_i^{\alpha}}{2\sigma^2}}\right)\left(1-\left(1-\ e^{-\left(\frac{\tau_i}{\sigma}\right)^{\alpha}}\right)^{\theta}\right)\right\} \tag{15}$$

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Rearranging the terms, equation (15) can be written as:

$$L = \prod_{t_i \in \omega_{11}} \left[\left(\alpha\, \lambda\, t_i^{\alpha-1}\; e^{-\lambda\, t_i^{\alpha}}\right) \left(e^{-\frac{t_i^{\alpha}}{2\sigma^2}}\right) \left(1 - \left(1 - e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)^{\theta}\right)\right]^{I\,(\delta_i = 1)}$$

$$* \prod_{t_i \in \omega_{12}} \left[\left(\frac{t_i}{\sigma^2}\; e^{-\frac{t_i^{\alpha}}{2\sigma^2}}\right) \left(e^{-\lambda\, t_i^{\alpha}}\right) \left(1 - \left(1 - e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)^{\theta}\right)\right]^{I\,(\delta_i = 2)}$$

$$* \prod_{t_i \in \omega_{13}} \left[\begin{array}{c} \left(\frac{\alpha\,\theta}{\sigma} \left(\frac{t_i}{\sigma}\right)^{\alpha-1} e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right) \left(1 - e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)^{\theta-1} \\ \left(e^{-\lambda\, t_i^{\alpha}}\right) \left(e^{-\frac{t_i^{\alpha}}{2\sigma^2}}\right) \end{array} \right]^{I\,(\delta_i = 3)}$$

$$* \prod_{t_i \in \omega_2} \left\{ \left[\left(e^{-\lambda\, t_i^{\alpha}}\; e^{-\frac{t_i^{\alpha}}{2\sigma^2}}\right) \left(\alpha\,\lambda\, t_i^{\alpha-1}\right) \left(1 - \left(1 - e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)^{\theta}\right) + \frac{t_i}{\sigma^2} \left(1 - \left(1 - e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)^{\theta}\right) + \left(\frac{\alpha\,\theta}{\sigma} \left(\frac{t_i}{\sigma}\right)^{\alpha-1} e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right) \left(1 - e^{-\left(\frac{t_i}{\sigma}\right)^{\alpha}}\right)^{\theta-1} \right]^{I\,(\delta_i = *)} \right\}$$

$$\prod_{t_i \in \omega_3} \left(e^{-\lambda\, \tau_i^{\alpha}}\, e^{-\frac{\tau_i^{\alpha}}{2\sigma^2}}\right) \left(1 - \left(1 - e^{-\left(\frac{\tau_i}{\sigma}\right)^{\alpha}}\right)^{\theta}\right) \qquad (16)$$

To estimate the unknown parameters, the first partial derivations of the log-likelihood function, $\log(\mathrm{L})$, with respect to α, θ, σ^2 and λ, respectively is needed.

Setting $\frac{\partial \log(\mathrm{L})}{\partial \alpha} = 0$, $\frac{\partial \log(\mathrm{L})}{\partial \theta} = 0$, $\frac{\partial \log(\mathrm{L})}{\partial \sigma^2} = 0$ *and* $\frac{\partial \log(\mathrm{L})}{\partial \lambda} = 0$,

we get the likelihood equations. These equations constitute a system of four nonlinear equations that must be solved in α, θ, σ^2 and λ to get the *MLEs* of these parameters. It is obvious that the system of nonlinear equations has no closed form solutions. So, a numerical technique is required to get the estimates of the unknown parameters.

5. THE RELATIVE RISK RATES

Relative risk (*RR*) is the ratio of the probability of an event occurring. Another term for the relative risk is the risk ratio because it is the ratio of the risk in the exposed divided by the risk in the unexposed. Kundu and Basu (2000) considered the relative risk rate due to two causes respectively as follows:

$$\pi_1 = \int_0^{\infty} f_1(t_i)\; S_2(t_i) \quad dt \qquad (17)$$

$$and \quad \pi_2 = \int_0^{\infty} f_2(t_i)\; S_1(t_i) \quad dt \qquad (18)$$

Similarly, as long as the three causes are independent, this paper introduced the relative risk rates due to the three causes, respectively, as follows:

$$\pi_W = \int_0^{\infty} f_1(t_i)\; S_2(t_i)\; S_3(t_i)\; dt, \qquad (19)$$

$$\pi_{EW} = \int_0^\infty f_2(t_i)\ S_1(t_i)\ S_3(t_i)\ dt \qquad (20)$$

$$and \quad \pi_R = \int_0^\infty f_3(t_i)\ S_1(t_i)\ S_2(t_i)\ dt \qquad (21)$$

These integrals which are mentioned in equations (19, 20 and 21) respectively have no closed analytical solution. So, numerical integration is required to get π_1, π_2 and π_3, respectively. The relative risks are obtained by replacing the unknown parameters in the above relations with their maximum likelihood estimates. Details of results are given from Table 1 through Table 8, in the Appendix.

6- SIMULATION STUDY

In this section, some simulation results to estimate the parameters included in three independent competing risks in the presence of complete observations, incomplete observations and censored observations according to *Type-II* censoring from some of *W*eibull models are presented. These results show the behavior of different sample sizes and also different parametric values. *MathCad-14* package is used to conduct the simulation study. The case when the competing risks (three causes) have *W* distribution, *EW* distribution and *R* distribution is considered. The behavior of the *MLEs* in terms of their biases and in terms of their variances is observed and the results are reported in Table 1 through Table 8 in the appendix. The results when censored observation obtained according to *Type-II* censoring were reported in Table 1 through Table 8. The simulation experiments are conducted according to the following steps:

1- The samples are drawn randomly for different values of *n*, *n= 25, 50* and *100,* for different parameters of *W* distribution, *EW* distribution and *R* distribution.

2- The values of different parameters were chosen on purpose by the author to ensure that there is an area to compete between the three causes as shown in Figure 1 through Figure 8.

3- The time of failures for the unknown cause was randomly predetermined.

4- According to the assumptions, for *Type-II* censoring, it is assumed that the experiment stopped at a fixed number of failure for each sample and for different parameters, namely *m.* it was chosen by the author, 20%, depending on the results of generated lifetimes for each sample size to ensure that there are enough observations to estimate the unknown parameters.

5- For the sample size n=100, as a special case, the percent of the number of censored data was changed, *m*= 10% , 20% and 40%, to observe the behavior of data forward to change the number of censoring.

6- The minimum lifetime value from the three distributions is selected to create a new sample.

7- The curves of the *p.d.f* , *c.d.*f and *h.f* of the *W* distribution, *EW* distribution and *R* distribution for the competing area with different values of the parameters are shown in Figure (1) through Figure (8).

8- The *MLEs* of α, θ, σ and λ for a new sample are computed.

9- The process are replicated ten thousand times, *N=10000.*

10- The average values of *MLEs*, their biases and the variance-covariance matrix are computed.

11- The average values of *RR* are obtained by replacing the unknown parameters in equations (19, 20 and 21) respectively with their maximum likelihood estimates.

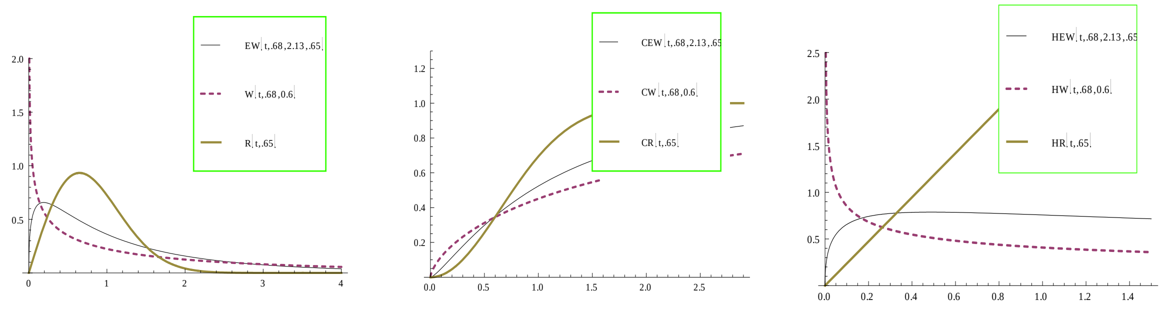

Fig. (1): Curves of the p.d.f (EW, W and R), c.d.f (CEW, CW and CR) and h.f.(HEW, HW and HR) for EW distribution at α =0.68, θ =2.13 and σ =0.65, W distribution at α =0.68, λ =0.6 and R distribution at σ = 0.65

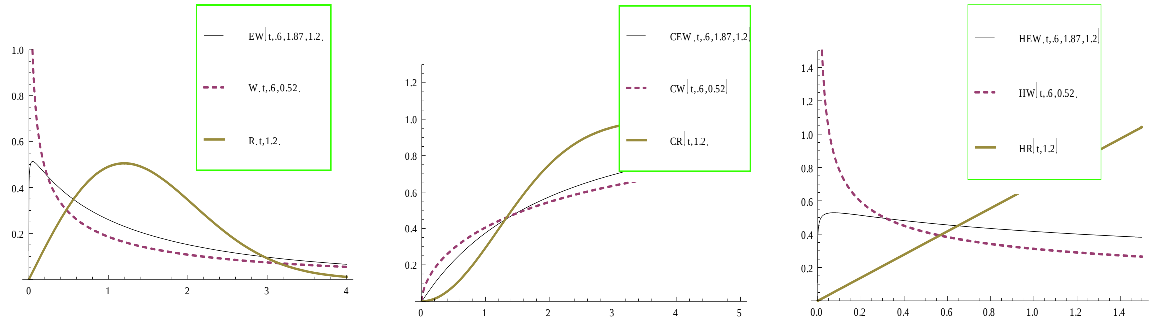

Fig. (2): Curves of the p.d.f (EW, W and R), c.d.f (CEW, CW and CR) and h.f.(HEW, HW and HR) for EW distribution at α =0.6, θ =1.87 and σ =1.2, W distribution at α =0.6, λ =0.52 and R distribution at σ =1.2

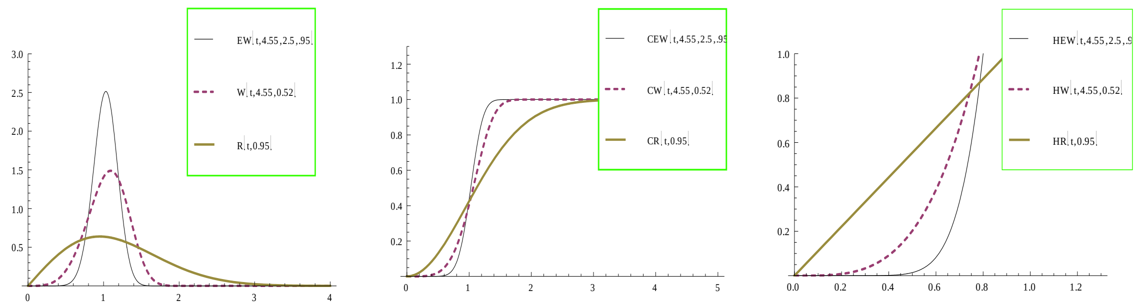

Fig. (3): Curves of the p.d.f (EW, W and R), c.d.f (CEW, CW and CR) and h.f.(HEW, HW and HR) for EW distribution at α =4.55, θ =2.5 and σ =0.95, W distribution at α =4.55, λ = 0.52and R distribution at σ =0.95

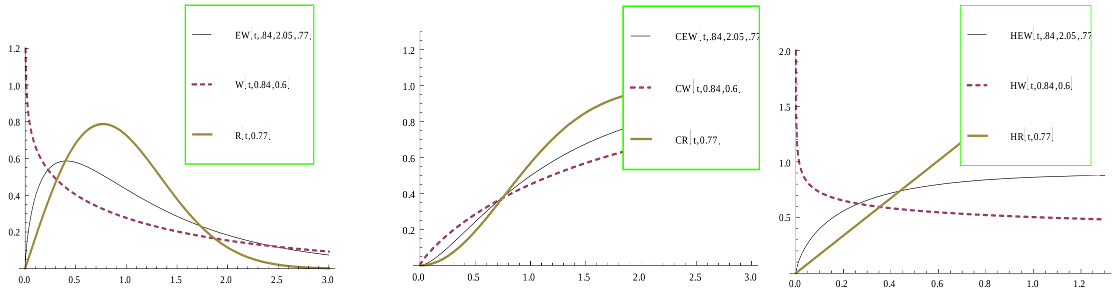

Fig. (4): Curves of the p.d.f (EW, W and R), c.d.f (CEW, CW and CR) and h.f.(HEW, HW and HR) for EW distribution at α =0.84, θ =2.05 and σ =0.77, W distribution at α =0.84, λ =0.6 and R distribution at σ =0.77

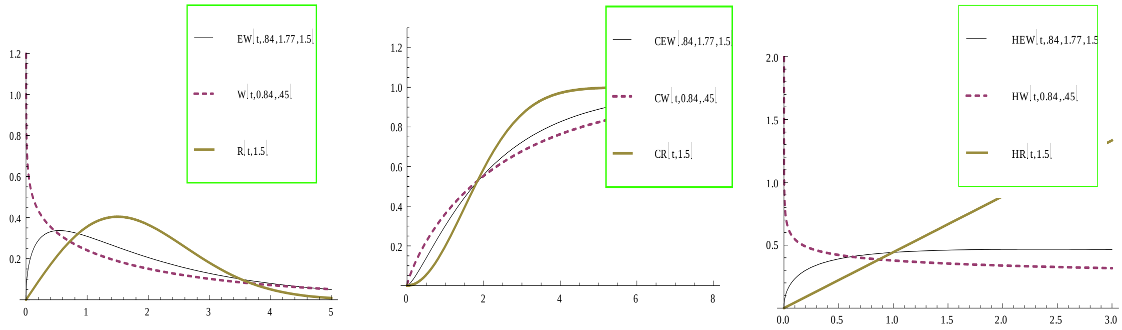

Fig. (5): Curves of the p.d.f (EW, W and R), c.d.f (CEW, CW and CR) and h.f.(HEW, HW and HR) for EW distribution at α =0.84, θ =1.77 and σ =1.5, W distribution at α =0.84, λ =0.45 and R distribution at σ =0.84

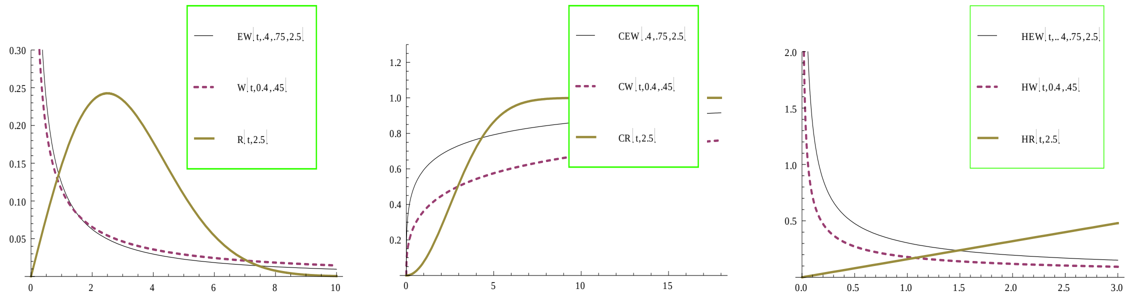

Fig. (6): Curves of the p.d.f (EW, W and R), c.d.f (CEW, CW and CR) and h.f.(HEW, HW and HR) for EW distribution at α =0.4, θ =0.75 and σ =2.5, W distribution at α =0.4, λ =0.45 and R distribution at σ =2.5

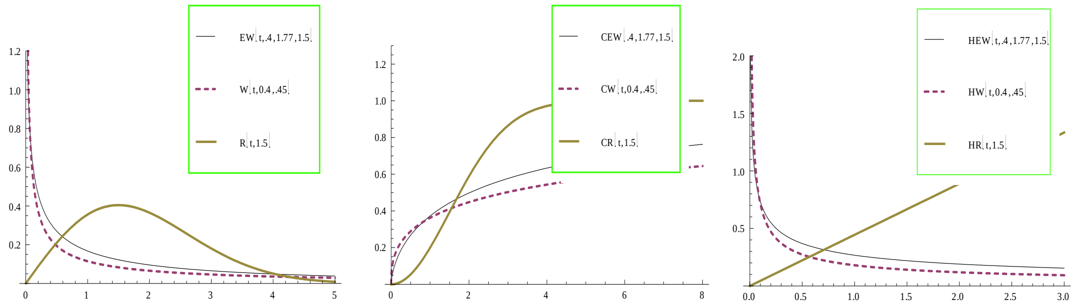

Fig. (7): Curves of the p.d.f (EW, W and R), c.d.f (CEW, CW and CR) and h.f.(HEW, HW and HR) for the EW distribution at α =0.4, θ =1.77 and σ =1.5, W distribution at α =0.4, λ =0.45 and R distribution at σ =1.5

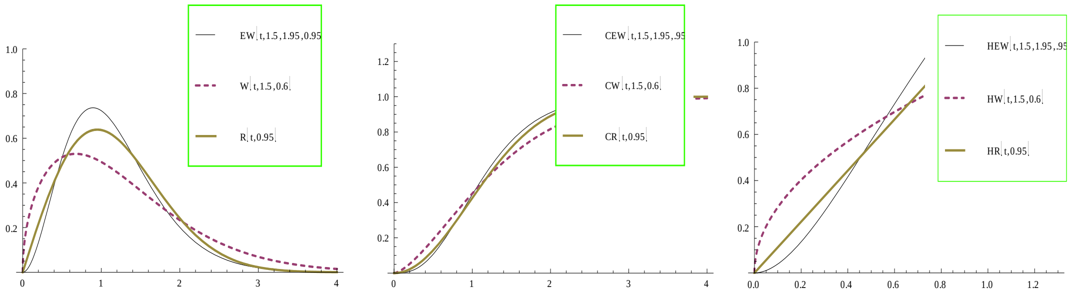

Fig. (8): Curves of the p.d.f (EW, W and R), c.d.f (CEW, CW and CR) and h.f.(HEW, HW and HR) for the *EW* distribution at α =1.5, θ =1.95and σ =0.95, W distribution at α =1.5, λ =0.6 and R distribution at σ =0.95

7- CONCLUDING REMARKS

In this paper, the estimators of the parameters included in independent competing risks with consideration mainly that there are three independent causes of failures are introduced. It is considered the case when the competing risks (three causes) have *W* distribution with two parameters, *EW* distribution and *R* distribution, respectively in the presence of complete observations, incomplete observations and *Type-II* censored data. Based on the results reported in Table 1 through Table 8 in the appendix, it is observed that:

1- The results of $\hat{\alpha}, \hat{\theta}, \hat{\sigma}$ *and* $\hat{\lambda}$ are approximately similar in nature.

2- It is clear that, as sample sizes increase the biases and the variance-covariance matrix decrease. This suggests that the *MLEs* are asymptotically unbiased and consistent estimators of the corresponding parameters.

3- Also, it is observed that as the sample size increases the theoretical relative risks due to cause 1, *W* distribution, and simulated relative risks are close to each other. The same comment applies for cause 2, *EW* distribution, and cause 3, *R* distribution respectively.

4- It should be emphasized that, the selected values of the parameters for generated samples which were selected according to the competing area are very important to consider.

5- According to the competing area, corresponding to our results, the best behavior of the hazard function appeared when the *W* distribution has a decreasing failure rate, as α <1. Also for the *EW* distribution, the best behavior of the hazard function appeared when $\alpha \le 1$ and $\alpha\,\theta \le 1$, decreasing failure rate, and when $\alpha < 1$ and $\alpha\,\theta > 1$, unimodal failure rate.

6- The choice of the number of censored data m is very important to consider, because the amount of this number is very affective on the number of observations for estimating the parameters.

7- For the case of sample size $n= 100$, as a special case, it is observed that the best estimate of the unknown parameters was appeared in the case when the least number of censored data is obtained, m=10. That remark indicated that, as the number of censored date increases, according to *Type-II* censoring, the biases and the variance-covariance matrix increase. That is not good fit for the estimation of the parameters which concerned with.

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APPENDIX

Table (1): The average values of $\hat{\alpha}, \hat{\theta}, \hat{\sigma}$ *and* $\hat{\lambda}$, their average bias, and covariances under *Type-II* censoring at α =0.68, θ =2.13, σ =0.65 and λ =0.6 where $\pi_W = 0.316$, $\pi_{EW} = 0.309$ and $\pi_R = 0.375$ (*W* distribution has decreasing failure rate and *EW* distribution has unimodal failure rate)

n	*m*	***Estimated value***				***Bias***				***R R***			***Variance-Covariance***
		$\hat{\alpha}$	$\hat{\theta}$	$\hat{\sigma}$	$\hat{\lambda}$					$\hat{\pi}_W$	$\hat{\pi}_{EW}$	$\hat{\pi}_R$	
25	5	0.97	2.08	0.499	1.25	0.29	-0.043	-0.151	0.65	0.283	0.413	0.304	$0.116 \quad -0.138 \quad 7.25E-3 \quad 0.134$ $0.885 \quad -0.035 \quad -0.145$ $0.01 \quad 0.011$ 0.436
50	10	0.88	2.097	0.471	1.095	0.174	-0.06	-0.165	0.396	0.361	0.303	0.336	$0.047 \quad -0.058 \quad 4.44E-3 \quad 0.027$ $0.37 \quad -0.023 \quad -0.038$ $4.8E-3 \quad 6.4E-3$ 0.114
100	10	0.781	2.144	0.525	0.79	0.101	0.014	-0.125	0.19	0.336	0.305	0.359	$0.012 \quad -0.025 \quad 1.421E-3 \quad 8.2E-3$ $0.183 \quad -7.04E-3 \quad -9.3E-3$ $1.7E-3 \quad 1.68E-3$ 0.033
	20	0.853	2.061	0.458	1.055	0.173	-0.069	-0.192	0.455	0.303	0.363	0.334	$0.019 \quad -0.034 \quad 2.271E-3 \quad 0.019$ $0.186 \quad -8.43E-3 \quad -0.031$ $2.42E-3 \quad 8.12E-4$ 0.073
	40	0.956	1.89	0.35	1.91	0.276	-0.235	-0.3	1.31	0.296	0.424	0.28	$0.026 \quad -0.038 \quad 2.67E-3 \quad 0.053$ $0.19 \quad -8.37E-3 \quad -0.084$ $2.14E-3 \quad 2.35E-3$ 0.306

Table (2): The average values of $\hat{\alpha}, \hat{\theta}, \hat{\sigma}$ *and* $\hat{\lambda}$, their average bias, and covariances under *Type-II* censoring at α =0.6, θ =1.87, σ =1.2 and λ =0.52 where $\pi_W = 0.371$, $\pi_{EW} = 0.318$ and $\pi_R = 0.311$ (*W* distribution has decreasing failure rate and *EW* distribution has unimodal failure rate)

n	*m*	***Estimated value***				***Bias***				***R R***			***Variance-Covariance***
		$\hat{\alpha}$	$\hat{\theta}$	$\hat{\sigma}$	$\hat{\lambda}$					$\hat{\pi}_W$	$\hat{\pi}_{EW}$	$\hat{\pi}_R$	
25	5	0.812	1.768	0.848	0.938	0.212	-0.102	-0.352	0.418	0.293	0.463	0.244	$0.066 \quad -0.103 \quad 0.013 \quad 0.056$ $0.466 \quad -.04 \quad -0.128$ $0.031 \quad 3.299E-3$ 0.189
50	10	0.755	1.852	0.781	0.862	0.155	-0.018	-0.419	0.342	0.298	0.429	0.273	$0.03 \quad -0.055 \quad 5.12E-3 \quad 0.023$ $0.257 \quad -0.021 \quad -0.036$ $0.017 \quad -8.34E-3$ 0.091
100	10	0.643	1.992	0.884	0.63	0.043	0.122	-0.316	0.11	0.301	0.379	0.32	$7.61E-3 \quad -0.013 \quad 7.73E-4 \quad 6.5E-3$ $0.093 \quad -9.601E-3 \quad -6.99E-3$ $5.16E-3 \quad -5.408E-4$ 0.018
	20	0.727	1.882	0.775	0.859	0.127	-0.048	-0.425	0.339	0.303	0.427	0.27	$0.011 \quad -0.015 \quad 1.43E-3 \quad 0.011$ $0.091 \quad -9.67E-3 \quad -0.015$ $5.63E-3 \quad 5.819E-4$ 0.039
	40	0.85	1.565	0.592	1.624	0.25	-0.305	-0.608	1.104	0.292	0.516	0.192	$0.014 \quad -0.022 \quad 3.14E-3 \quad 0.025$ $0.111 \quad -0.017 \quad -0.038$ $9.51E-3 \quad 2.35E-3$ 0.156

Table (3): The average values of $\hat{\alpha}, \hat{\theta}, \hat{\sigma}$ *and* $\hat{\lambda}$, their average bias, and covariances under *Type-II* censoring at α =4.55, θ =2.5, σ =0.95 and λ =0.52 where $\pi_W = 0.3$, $\pi_{EW} = 0.327$ and $\pi_R = 0.373$ (*W* distribution has increasing failure rate and *EW* distribution has increasing failure rate)

n	*m*	*Estimated value* $\hat{\alpha}$	$\hat{\theta}$	$\hat{\sigma}$	$\hat{\lambda}$	*Bias*				*R R* $\hat{\pi}_W$	$\hat{\pi}_{EW}$	$\hat{\pi}_R$	*Variance-Covariance*
25	5	5.434	50558	0.83	0.963	0.884	3.058	-0.12	0.443	0.253	0.334	0.413	2.481 -1.778 0.072 0.407 22.231 -0.247 -0.044 $6.86E-3$ $6.93E-3$ 0.275
50	10	5.336	5.187	0.819	0.989	0.786	2.687	-0.131	0.469	0.264	0.323	0.412	1.115 -1.173 0.029 0.224 10.026 0.135 -0.239 $3.44E-3$ $3.83E-3$ 0.161
100	10	4.595	4.247	0.843	0.642	0.045	1.747	-0.107	0.122	0.306	0.277	0.417	0.452 -0.533 0.02 0.047 2.458 -0.05 -0.079 $1.74E-3$ $2.15E-3$ 0.028
	20	5.295	4.562	0.817	1.005	0.745	2.062	-0.133	0.485	0.277	0.315	0.408	1.087 -1.25 0.038 0.197 5.038 -0.247 -0.296 $2.23E-3$ $6.93E-3$ 0.111
	40	5.453	7.039	0.727	1.899	0.903	4.539	-0.223	1.379	0.237	0.341	0.422	1.51 -2.129 0.057 0.846 13.127 -0.145 -1.087 $3.365E-3$ 0.024 0.865

Table (4): The average values of $\hat{\alpha}, \hat{\theta}, \hat{\sigma}$ *and* $\hat{\lambda}$, their average bias, and covariances under *Type-II* censoring at α =0.84, θ =2.05, σ =0.77 and λ =0.6 where $\pi_W = 0.332$, $\pi_{EW} = 0.317$ and $\pi_R = 0.351$ (*W* distribution has decreasing failure rate and *EW* distribution has unimodal failure rate)

n	*m*	*Estimated value* $\hat{\alpha}$	$\hat{\theta}$	$\hat{\sigma}$	$\hat{\lambda}$	*Bias*				*R R* $\hat{\pi}_W$	$\hat{\pi}_{EW}$	$\hat{\pi}_R$	*Variance-Covariance*
25	5	1.192	2.051	0.6	1.225	0.352	0.001	- 0.17	0.625	0.284	0.418	0.298	0.16 -0.18 0.018 0.137 0.827 -0.052 -0.109 0.016 $-2.88E-3$ 0.414
50	10	1.129	2.049	0.568	1.128	0.289	-0.001	- 0.202	0.528	0.298	0.387	0.315	0.078 -0.082 $3.28E-3$ 0.053 0.342 -0.024 -0.06 $7.65E-3$ $-7.75E-4$ 0.149
100	10	0.95	2.103	0.626	0.8	0.11	0.053	- 0.144	0.2	0.306	0.35	0.344	0.018 -0.032 $3.47E-3$ $9.52E-3$ 0.141 $-9.64E-3$ -0.015 $3.31E-3$ $7.16E-4$ 0.03
	20	1.05	2.017	0.551	1.094	0.21	-0.033	-0.219	0.494	0.303	0.381	0.316	0.029 -0.046 $5.21E-3$ 0.027 0.158 -0.011 -0.04 $3.54E-3$ $2.68E-3$ 0.082
	40	1.201	1.9	0.425	2.134	0.361	-0.15	-0.345	1.534	0.284	0.447	0.269	0.054 -0.069 $6.35E-3$ 0.125 0.186 -0.013 -0.158 $3.68E-3$ $8.18E-3$ 0.556

Table (5): The average values of $\hat{\alpha}, \hat{\theta}, \hat{\sigma}$ *and* $\hat{\lambda}$, their average bias, and covariances under *Type-II* censoring at α =0.68, θ =1.77, σ =1.5 and λ =0.45 where $\pi_W = 0.388$, $\pi_{EW} = 0.327$ and $\pi_R = 0.285$ (*W* distribution has decreasing failure rate and *EW* distribution has unimodal failure rate)

n	*m*	*Estimated value*				*Bias*				*R R*			*Variance-Covariance*
		$\hat{\alpha}$	$\hat{\theta}$	$\hat{\sigma}$	$\hat{\lambda}$					$\hat{\pi}_W$	$\hat{\pi}_{EW}$	$\hat{\pi}_R$	
25	5	0.88	1.866	1.016	0.674	0.2	0.096	-0.484	0.224	0.303	0.416	0.281	$0.084\ \ -0.119\ \ 1.91E-3\ \ 0.02$ $0.695\ \ -0.053\ \ -0.011$ $0.036\ \ -0.019$ 0.097
50	10	0.843	1.808	0.973	0.658	0.163	0.038	-0.527	0.2082	0.319	0.395	0.286	$0.04\ \ -0.059\ \ 3.79E-3\ \ 8.36E-3$ $0.211\ \ -0.021\ \ -0.027$ $0.022\ \ -4.27E-3$ 0.043
100	10	0.684	1.96	1.068	0.474	0.0037	0.19	-0.432	0.024	0.323	0.335	0.343	$7.69E-3\ \ -0.018\ \ 1.82E-3\ \ 1.6E-3$ $0.106\ \ -6.28E-3\ \ -9.08E-4$ $8.76E-3\ \ -1.461E-3$ 0.01
	20	0.776	1.807	0.93	0.638	0.096	0.037	-0.57	0.188	0.326	0.378	0.296	$0.014\ \ -0.025\ \ 3.26E-3\ \ 5.97E-3$ $0.107\ \ -6.97E-3\ \ -7.69E-3$ $7.87E-3\ \ -2.56E-3$ 0.024
	40	0.98	1.455	0.745	1.337	0.302	-0.315	-0.755	0.887	0.316	0.49	0.195	$0.033\ \ -0.043\ \ 8.24E-3\ \ 0.036$ $0.108\ \ -0.015\ \ -0.046$ $0.015\ \ 2.05E-3$ 0.151

Table (6): The average values of $\hat{\alpha}, \hat{\theta}, \hat{\sigma}$ *and* $\hat{\lambda}$, their average bias, and covariances under *Type-II* censoring at α =0.4, θ =0.75, σ =2.5 and λ =0.45 where $\pi_W = 0.275$, $\pi_{EW} = 0.573$ and $\pi_R = 0.152$ (*W* distribution and *EW* distribution have decreasing failure rate)

n	*m*	*Estimated value*				*Bias*				*R R*			*Variance-Covariance*
		$\hat{\alpha}$	$\hat{\theta}$	$\hat{\sigma}$	$\hat{\lambda}$					$\hat{\pi}_W$	$\hat{\pi}_{EW}$	$\hat{\pi}_R$	
25	5	0.641	0.613	1.301	1.02	0.241	-0.137	-1.199	0.57	0.541	0.376	0.084	$0.112\ \ -0.066\ \ 0.024\ \ 0.079$ $0.118\ \ -0.018\ \ -0.075$ $0.305\ \ -0.072$ 0.48
50	10	0.592	0.589	1.096	1.042	0.192	-0.161	-1.404	0.592	0.567	0.347	0.087	$0.041\ \ -0.029\ \ -7.82E-3\ \ 0.122$ $0.033\ \ -9.15E-5\ \ -0.072$ $0.096\ \ -0.059$ 0.91
100	10	0.443	0.725	1.409	0.563	0.043	-0.025	-1.091	0.113	0.58	0.272	0.147	$3.5E-3\ \ -5.6E-3\ \ -2.6E-3\ \ 2.9E-3$ $0.015\ \ 3.7E-4\ \ -5.3-3$ $0.044\ \ -8.6E-3$ 0.019
	20	0.547	0.574	1.07	0.906	0.147	-0.176	-1.43	0.456	0.596	0.308	0.096	$0.011\ \ -0.01\ \ 8.8E-4\ \ 0.016$ $0.015\ \ -5.31E-4\ \ -0.016$ $0.047\ \ -0.21$ 0.084
	40	0.683	0.455	0.495	2.518	0.283	-0.295	-2.005	2.068	0.584	0.364	0.052	$0.015\ \ -8.9E-3\ \ -3.9E-4\ \ 0.048$ $9.6E-3\ \ -5.6E-3\ \ -0.026$ $0.059\ \ -0.073$ 0.663

Table (7): The average values of $\hat{\alpha}, \hat{\theta}, \hat{\sigma}$ *and* $\hat{\lambda}$, their average bias, and covariances under *Type-II* censoring at α =0.4, θ =1.77, σ =1.5 and λ =0.45 where $\pi_W = 0.34$, $\pi_{EW} = 0.341$ and $\pi_R = 0.319$ (*W* distribution and *EW* distribution have decreasing failure rate)

n	*m*	*Estimated value*				*Bias*				*R R*			*Variance-Covariance*
		$\hat{\alpha}$	$\hat{\theta}$	$\hat{\sigma}$	$\hat{\lambda}$					$\hat{\pi}_W$	$\hat{\pi}_{EW}$	$\hat{\pi}_R$	
25	5	0.526	1.656	0.976	0.694	0.126	-0.124	-0.524	0.244	0.336	0.403	0.261	0.03 -0.056 $-1.7E-3$ 0.02 0.354 -0.019 -0.02 0.046 -0.019 0.105
50	10	0.506	1.629	0.933	0.688	0.106	-0.141	-0.567	0.238	0.342	0.393	0.265	0.014 -0.03 $-1.3E-3$ $7.3E-3$ 0.157 -0.012 -0.024 0.025 $-7.2E-3$ 0.51
100	10	0.418	1.785	1.074	0.507	0.018	0.015	-0.426	0.057	0.34	0.338	0.322	$2.9E-3$ -0.01 $5.05E-4$ $1.5E-3$ 0.084 $-3.8E-3$ $-6.35E-3$ $9.9E-3$ $-1.6E-3$ 0.011
	20	0.472	1.613	0.894	0.675	0.072	-0.157	-0.606	0.225	0.349	0.381	0.269	$5.02E-3$ -0.014 $6.2E-4$ $4.3E-3$ 0.079 $-1.75E-3$ -0.014 $8.3E-3$ $-3.5E-3$ 0.025
	40	0.595	1.272	0.633	1.402	0.195	-0.498	-0.867	0.952	0.348	0.494	0.158	0.013 -0.025 $3.1E-3$ 0.025 0.082 $-8.9E-3$ -0.055 0.016 $-3.9E-3$ 0.169

Table (8): The average values of $\hat{\alpha}, \hat{\theta}, \hat{\sigma}$ *and* $\hat{\lambda}$, their average bias, and covariances under *Type-II* censoring at α =1.5, θ =1.95, σ =0.95 and λ =0.6 where $\pi_W = 0.36$, $\pi_{EW} = 0.323$ and $\pi_R = 0.317$ (*W* distribution and *EW* distribution have increasing failure rate)

n	*m*	*Estimated value*				*Bias*				*R R*			*Variance-Covariance*
		$\hat{\alpha}$	$\hat{\theta}$	$\hat{\sigma}$	$\hat{\lambda}$					$\hat{\pi}_W$	$\hat{\pi}_{EW}$	$\hat{\pi}_R$	
25	5	2.114	2.4	0.755	1.215	0.614	0.45	- 0.195	0.615	0.252	0.427	0.321	0.496 -0.425 0.029 0.273 2.423 -0.098 -0.099 0.017 $-6.52E-3$ 0.518
50	10	2.02	2.268	0.734	1.147	0.52	0.318	- 0.216	0.547	0.276	0.399	0.325	0.239 -0.166 $9.4E-3$ 0.085 0.851 -0.053 -0.076 $8.88E-3$ $-4.2E-3$ 0.154
100	10	1.657	2.241	0.784	0.784	0.157	0.291	- 0.166	0.184	0.299	0.36	0.341	0.052 -0.058 $7.5E-3$ 0.017 0.206 -0.018 -0.02 $3.6E-3$ $1.6E-3$ 0.03
	20	1.894	2.097	0.724	1.123	0.394	0.147	-0.226	0.523	0.294	0.388	0.317	0.099 -0.102 0.11 0.044 0.232 -0.021 -0.056 $4.35E-3$ $2.8E-3$ 0.084
	40	2.117	2.23	0.584	2.203	0.617	0.28	-0.366	1.603	0.26	0.431	0.309	0.18 -0.15 0.13 0.235 0.399 -0.031 -0.202 $5.06E-3$ $8.07E-3$ 0.653